# Combination

- Combination is the selection of different items from a given number of items
- Combination is order independent
- The total number of combinations or selections of r items from n different items will be given by;

  ${}^nC_r$ or C(n, r) = $\frac{\lfloor n}{\lfloor r \times \lfloor n-r}$ where r $\leq n$

- No arrangement (Permutation) is possible without selection (Combination) but selection (Combination) process can take place independently
- Thus ${}^nC_r < {}^nP_r$, except when r=0 or 1
- Relation between ${}^nP_r$ and ${}^nC_r$
- ${}^nP_r = {}^nC_r \times \lfloor r$
- $\frac{{}^nP_r}{{}^nC_r} = \lfloor r$
- ${}^nC_0 = {}^nP_0 = 1$
- ${}^nC_1 = {}^nP_1 = n$
- ${}^nC_n = 1, {}^nP_n = \lfloor n$

**Complementary Combination**

- ${}^nC_r = {}^nC_{n-r}$ (Use this result, when $r > \frac{n}{2}$)
- If ${}^nC_x = {}^nC_y$, then either
  a. x = y or
  b. x+y=n or
  c. both the results can hold true simultaneously
- ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
- $\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$
- The total number of selection from n items by taking 1 or more (at least one), will be given by;
  ${}^nC_1 + {}^nC_2 + {}^nC_3 + ..... + {}^nC_n = 2^n - 1$

**Application of combinations**

**Case 1:**

Total number of straight lines that can be formed out of 'n' points on a plane when no **three** of them are co-linear will be given by ${}^{n}C_{2}$

**Case 2:**

Total number of triangles that can be formed out of these 'n' points is ${}^{n}C_{3}$

**Case 3:**

Total Number of circles that can be formed out of these 'n' points is ${}^{n}C_{3}$

**Case 4:**

Total number of lines that can be formed with 'n' points when p of them are collinear will be given ${}^{n}C_{2} - {}^{p}C_{2} + 1$

**Case 5:**

Total number of triangles that can be formed with 'n' points when p of them are collinear will be given by ${}^{n}C_{3} - {}^{P}C_{3}$

**Case 6:**

The total number of points of intersection that can be obtained from 'n' straight lines are ${}^{n}C_{2}$ when,

i. No two of them are parallel and

**ii.** No **three** of them are concurrent

**Case 7:**

To find the number of diagonals in a polygon having 'n' sides

No of diagonals = ${}^{n}C_{2} - n$

Where ${}^{n}C_{2}$ = total number of lines by joining 2 vertices in pairs and

'n' number of sides = number of vertices

${}^{n}C_{2} - n$

$$= \frac{n(n-1)}{2} - n = n\left(\frac{n-1}{2} - 1\right) \Rightarrow n\left(\frac{n-1-2}{2}\right) \Rightarrow n\left(\frac{n-3}{2}\right)$$

**Case 8:**

Total number of selections or combinations of 'n' different things taking one or more at a time (i.e., at least 1) will be given by ${}^nC_1 + {}^nC_2 + {}^nC_3 + ..... + {}^nC_n$

$${}^nC_1 + {}^nC_2 + {}^nC_3 + ..... + {}^nC_n = 2^n - 1$$

**Case 9:**

**Combinations or selections of things which are alike**.

Total number of combinations or selection of p, q, r items by taking one or more(atleast one) will be given by, **(p+1)(q+1)(r+1)-1**

When p are alike and of one kind, q are alike and of a second kind and r are alike and of yet of another kind.

**Note:**

Total number of selections of p alike, q alike and r different items by taking at least one will be given by $(p+1)(q+1)2^r-1$

**Case 10: Division into groups**

- The total number of ways in which (m+n) items can be divided into two distinct groups containing m & n items respectively will be given by: $\frac{\lfloor m+n}{\lfloor m \lfloor n}$
- Total ways in which m+n+p items can be divided into 3 distinct groups containing m, n & p items respectively will be given by,
- $\frac{\lfloor m+n+p}{\lfloor m \lfloor n \lfloor p}$

**Case i:**

When m = n or m=n=p then 2m or 3m items can be equally distributed into two or three distinct groups in, $\frac{\lfloor 2m}{(\lfloor m)^2}$ or $\frac{\lfloor 3m}{(\lfloor m)^3}$ *ways*

**Case ii:**

When the identities of the groups are not distinct i.e, the groups are alike in such a case 2m or 3m items can be distributed equally into 2 or 3 identified groups in $\frac{\lfloor 2m}{(\lfloor m)^2} \times \frac{1}{2!}$ or $\frac{\lfloor 3m}{(\lfloor m)^3} \times \frac{1}{3!} ways$

## Basic Meaning

1. Find the value of n when $^{20}C_{3n} = {}^{20}C_{2n+5}$

   a) 3

   b) 5

   c) Both a) and b) above

   d) Neither a) nor b) above

2. If $^{n}C_{10} = {}^{n}C_{15,}$ find the value of $^{27}C_{n}$.

   a) 80730

   b) 40146

   c) 40376

   d) 351

3. If $^{15}C_{3r} = {}^{15}C_{r+3,}$ find the value of r.

   a) 3

   b) 4

   c) Both a) and b) above

   d) None of a) or b) above

4. If $^{2n}C_{r} = {}^{2n}C_{r+2,}$ find the value of r.

   a) n + 1

   b) n – 1

   c) 2n + 1

   d) None of the above

5. If ${}^{18}C_r = {}^{18}C_{r+2}$, find the value of r.

   a) 4

   b) 7

   c) 8

   d) 10

6. If ${}^{n}C_{12} = {}^{n}C_{10}$, what is the value of ${}^{n}C_2$?

   a) 222

   b) 231

   c) 321

   d) 259

7. If ${}^{n}P_r = 336$ and ${}^{n}C_r = 56$, find n and r. Find also ${}^{n+2}C_{r+1}$

   a) 8, 3, 120

   b) 8, 3, 210

   c) 8, 5, 210

   d) None of the above

8. Find the value of: ${}^{12}C_4 + {}^{12}C_3$

   a) 720

   b) 715

   c) 815

   d) 820

9. If ${}^{13}C_6 + 2.\ {}^{13}C_5 + {}^{13}C_4 = {}^{15}C_x$, what is the value of X?

   a) 6

   b) 9

   c) Either a) or b)

   d) Both a) and b)

10. If ${}^{1000}C_{98} = {}^{999}C_{97} + {}^{x}C_{901,}$ find x

    a) 1000

    b) 998

    c) 997

    d) 999

11. If ${}^{n}P_r = 6.{}^{n}C_r$, find n and r when ${}^{n}C_r = 56$.

    a) 8, 4

    b) 8, 5

    c) 8, 3

    d) All of the above

12. Find the value of $^{47}C_4 + \sum_{r=1}^{5} {}^{52-r}C_3$

    a) ${}^{52}C_6$

    b) ${}^{52}C_5$

    c) ${}^{52}C_4$

    d) None

13. If, $\frac{{}^{n}C_{r-1}}{{}^{n}C_r} = \frac{1}{4}$ **and** $\frac{{}^{n}C_r}{{}^{n}C_{r+1}} = \frac{1}{3}$ then find the value of n and r?

    a) 35,7

    b) 53.8

    c) 358

    d) 19.4

14. If ${}^{2n}C_3 : {}^{n}C_2 = 12 : 1$, find the value of n.

    a. 4

    b. 5

    c. 6

    d. 7

**<u>"n" different things, "r" to be selected (r ≤ n) – Without Restrictions</u>**

15.How many different committees of 5 members may be formed from 8 Indians and 4 Foreigners?

    a. 972

    b. 792

    c. 297

    d. 279

16. In how many ways can a committee of 6 men and 2 women be formed out of 10 men and 5 women?

a. 1200

b. 2100

c. 1300

d. 3100

17. A committee consists of 10 gentlemen and 8 ladies. How many different subcommittees can be formed consisting of 5 gentlemen and 3 ladies?

a. 14000

b. 14110

c. 14112

d. 11214

**"n" different things, "r" to be selected (r ≤ n) – With Restrictions**

In how many ways can four students be selected out of twelve students if

18. 2 particular students are not included at all.

a. 120
b. 210
c. 340
d. 320

19. 2 particular students are always included?

a. 54
b. 56
c. 57
d. 45

20. In how many ways can 7 men be selected from 16 men so that (i) 4 particular men will not be there, (ii) 4 particular men will always be there?

a. 792, 220

b. 792, 240

c. 780, 220

d. 820, 220

21. From 6 gentlemen and 4 ladies, a committee of 5 is to be formed. In how many ways can this be done so as to include at least one lady?

    a. 642

    b. 259

    c. 246

    d. 586

In how many ways can a committee of 5 members be formed from 10 candidates so as to

22. Include both the youngest and the oldest candidates

    a. 196

    b. 65

    c. 54

    d. 56

23. Exclude the youngest if it includes the oldest?

    a. 196

    b. 165

    c. 156

    d. 157

24. In how many ways can a committee of 5 be formed from 4 professors and 6 students so as to include at least 2 professors?

    a. 246

    b. 986

    c. 187

    d. 186

25. In how many ways can a committee of 3 ladies and 4 gentlemen be appointed from a meeting consisting of 8 ladies and 7 gentlemen? What will be the number of ways if Mrs. X refuses to serve in a committee if Mr. Y is a member?

    a. 1960, 1440

    b. 1960, 1540

    c. 1440, 1240

    d. 1540, 1440

26. In a class of 16 students, there are 5 lady students. In how many ways can 10 students be selected from them so as to include at least 4 lady students?
    a. 2770
    b. 2772
    c. 7227
    d. 7272

27. A certain council consists of a chairman, two vice-chairmen and twelve other members. How many different committees of 6 can be formed including always the chairman and only one vice-chairman?
    a. 1000
    b. 900
    c. 990
    d. 975

28. The question paper on Mathematics and Statistics contains 10 questions divided into two groups of 5 questions each. In how many ways can an examinee select 6 questions taking at least two questions from each group?
    a. 200
    b. 150
    c. 100
    d. 250

29. In an examination paper there are two groups each containing 7 questions. A candidate is required to attempt 9 questions but not more than five questions from any group. In how many ways can 9 questions be selected?
    a) 1470
    b) 1570
    c) 1680
    d) 1970

30. A candidate is required to answer 6 out of 12 questions which are divided into two groups each containing 6 questions and he is permitted to attempt not more than four from any group. In how many different ways can he make up his choice?
    a. 580
    b. 680
    c. 850
    d. 950

31. In an examination paper on Business Mathematics, there are 14 questions divided into three groups of 5, 5 and 4 questions respectively. A candidate is required to answer 6 questions taking at least two questions from each of the first two groups and one question from the third group. In how many different ways can he make up his choice?
    a. 1100
    b. 1200
    c. 1300
    d. 1400

32. A question paper contains 12 questions, of which 7 are in Group A and 5 in Group B. The questions are serially numbered from 1 to 12. If a candidate has to answer the fourth question and 3 other from Group A, and to answer eighth question and 2 other from Group B, in how many different ways can the candidate make up his choice?
    a. 110
    b. 115
    c. 119
    d. 120

## "n" different things, any number can be selected at a time

33. In how many ways can a person choose one or more of the four electrical appliances: T.V., Refrigerator, Washing Machine, Radiogram?
    a. 16
    b. 31
    c. 32
    d. 15

34. A man has 8 friends. In how many ways may be invite one or more of them to a dinner?
    a. 256
    b. 254
    c. 255
    d. 128

35. In how many ways a man can invite 5 friends to a dinner so that two or more of them remain present?
    a. 24
    b. 25
    c. 26
    d. 32

36. At an election there are 5 candidates and 3 members are to be elected and a voter is entitled to vote for any number to be elected. In how many ways may a voter choose a vote?

    a. 24

    b. 23

    c. 26

    d. 25

A man has 5 German, 4 Spanish and 3 French friends. Find:

37. Total ways in which all the friends can be invited.

    a. 4096

    b. 4095

    c. 2048

    d. None of the above

38. Total ways of invitation so that there is at least 1 German friend.

    a. $(2^5-1).2^4$

    b. $(2^5-1).2^3$

    c. $(2^5-1).2^7$

    d. None of the above

39. At least 1 German and 1 French friend.

    a. $(2^5-1).(2^4-1).2^3$

    b. $(2^5-1).(2^3-1).2^4$

    c. $(2^4-1).2^3.2^5$

    d. None of the above

40. At least one friend from each country.

    a. 3165

    b. 3290

    c. 3255

    d. 3250

41. Ten electric bulbs, of which 3 are defective, are to be tried in three different light points in a dark room. In how many out of the total trial the room shall be lighted?
    a. 120
    b. 121
    c. 119
    d. 122

42. Five bulbs of which three are defective are to be tried in two light points in a dark room. In how many trials the room shall be lighted?
    a. 7
    b. 8
    c. 9
    d. 11

43. There are 10 lamps in a hall. Each of them can be switched on independently. What is the number of ways in which the hall can be illuminated?
    a. 1024
    b. 100
    c. 1023
    d. 120

44. A Supreme Court Bench consists of five judges. In how many ways the bench can give a majority decision?
    a. 16
    b. 15
    c. 31
    d. 32

45. Taking data from the previous question; also calculate the ways of negation not affecting the majority decision.
    a. 31
    b. 32
    c. 15
    d. 16

**Application of Combination in Geometry**

46. How many straight lines can be obtained by joining 16 points on a plane, no three points being on the same line?

    a. 120

    b. 240

    c. 119

    d. 480

47. How many triangles can be drawn, by joining 16 points on a plane, no three points being on the same line?

    a. 240

    b. 360

    c. 560

    d. 480

48. Find the number of straight lines formed by joining 10 different points on a plane, no three of them being collinear (with the exception of 4 points which are collinear).

    a. 41

    b. 45

    c. 39

    d. 40

49. Find the number of triangles formed by joining 10 different points on a plane, no three of them being collinear (with the exception of 4 points which are collinear).

    a. 120

    b. 116

    c. 121

    d. 126

50. If 20 straight lines be drawn in a plane, no two of them being parallel and no three of them being concurrent, how many points of intersection will there be?

    a. 200

    b. 180

    c. 210

    d. 190

Find the number of diagonals in:

51. Quadrilateral

a. 2

b. 3

c. 4

d. 6

52. Octagon

a. 10

b. 35

c. 20

d. 9

53. Decagon

a. 35

b. 20

c. 15

d. 25

54. A polygon has 44 diagonals. Find the number of its sides.

a. 10

b. 11

c. 12

d. 14

## Selections of p,q,r alike items taken any numbere at a time

There are 8 mangoes, 4 apples and 5 oranges. If any number of fruits can be selected, find:

55. Total number of selections.

a. 269

b. 270

c. 275

d. 300

56. Total number of selections so that there is at least 1 mango.

    a. 269

    b. 270

    c. 240

    d. None

57. At least 1 mango and 1 apple.

    a. 129

    b. 108

    c. 40

    d. 192

58. At least 1 fruit of each type.

    a. 160

    b. 170

    c. 190

    d. 210

How many selections can be made by taking any letters from the words

59. DADDY DID A DEADLY DEED

    a. 1920

    b. 1921

    c. 1290

    d. 1919

60. SHE SELLS SEA SHELLS

    a. 1049

    b. 1050

    c. 1079

    d. 1080

61. There are "n" different video films and "p" cassettes of each film in a video library. Find the number of ways in which one or more than one video cassette can be selected?
    a) $(p+1)^n$
    b) $(p + 1)$
    c) $(p + 1)^n - 1$
    d) None of the above

62. A person has in his bag 14 notes of Rs. 10 each, 9 notes of Rs. 5 each, 4 notes of Rs. 2 each and 7 notes of Re. 1 each. In how many different ways can he contribute to a charitable fund?
    a) 3000
    b) 6000
    c) 5999
    d) 2999

## Division into Groups – either distinct or alike

63. The three different sections of a Library need the services of 3, 4 and 5 workers respectively. If 12 workers are available, in how many ways can they be allotted to different sections?
    a. 27720
    b. 22770
    c. 27270
    d. 72270

64. Divide 12 items in two groups so that each containing 8 and 4 items.
    a) $\frac{12!}{8!}$
    b) $\frac{12!}{4!8!}$
    c) $\frac{8!4!}{12!}$
    d) None of the above

<u>In how many ways can 20 books be equally divided amongst:</u>

65. 2 Boys.
    a. $\frac{20!}{(10!)^2}$
    b. $\frac{20!}{(5!)^4}$
    c. $\frac{20!}{(4!)^5}$
    d. $\frac{20!}{(2!)^{10}}$

66. 4 Boys.

a. $\frac{20!}{(10!)^2}$

b. $\frac{20!}{(5!)^4}$

c. $\frac{20!}{(4!)^5}$

d. $\frac{20!}{(2!)^{10}}$

67. 5 Boys.

a. $\frac{20!}{(10!)^2}$

b. $\frac{20!}{(5!)^4}$

c. $\frac{20!}{(4!)^5}$

d. $\frac{20!}{(2!)^{10}}$

68. 10 Boys.

a. $\frac{20!}{(10!)^2}$

b. $\frac{20!}{(5!)^4}$

c. $\frac{20!}{(4!)^5}$

d. $\frac{20!}{(2!)^{10}}$

69. In how many ways 9 students be divided equally into 3 groups?
    a) 260
    b) 270
    c) 280
    d) 300

**"n" things, few are alike, "r" things chosen at a time and then arranged**

How many different words can be formed taking 4 letters from the following words?

70. PROPORTION
    a. 758
    b. 875
    c. 587
    d. 578

71. EXAMINATION
    a. 2434
    b. 2454
    c. 1424
    d. 6424

**Mixed Bag**

72. There are 25 candidates, which include 5 from the Scheduled Castes for 12 vacancies. If 3 vacancies are reserved for Scheduled Caste candidates and the remaining vacancies are open to all, find the number of ways in which the selection can be made
    a. ${}^{5}C_{3}.{}^{20}C_{9}$
    b. ${}^{5}C_{3}.{}^{22}C_{9}$
    c. ${}^{25}C_{12}$
    d. None

73. A league match in football ends in a win, loss or draw. How many different forecasts can give exactly 18 correct results out of 22 matches played?
    a. ${}^{22}C_{18}$
    b. ${}^{22}C_{18}.2^{4}$
    c. ${}^{22}C_{18}.4^{2}$
    d. None

74. In how many ways can the letters of the word FORECAST taken 3 at a time and the word MILKY taken 2 at a time be arranged?

a. 62700

b. 67000

c. 68720

d. 67200

75. A “number lock” consists of 3 rings each marked with 10 different numbers. In how many ways the lock cannot be opened?

a) 1000

b) 999

c) 998

d) 997

76. How many different factors can 210 have?

a) 15

b) 16

c) 31

d) 32

77. How many different factors can 2160 have?

a) 40

b) 39

c) 37

d) 45

78. A captain and a vice-captain are to be chosen out of a team having 11 players. How many ways are there to achieve this?

a) 110

b) 110

c) 120

d) None of the above

79. A plane can accommodate 250 passengers, 140 in the economy class and 110 in the business class. In how many ways can 250 passengers be accommodated if 25 refuses to sit in economy class and 20 cannot afford the business class?

a. $\frac{205!}{85! \times 120!}$

b. $\frac{205! \times 25! \times 85!}{120! \times 140!}$

c. $\frac{205!}{140! \times 110!}$

d. $\frac{205! \times 140! \times 110!}{120! \times 85!}$

80. 18 guests are to be arranged along two sides of a long rectangular table, nine on each side. Four articular guests which to sit on one side and three on the other side. In how many ways can all the guests be arranged?