CHAPTER 12
Limits and Continuity

Limits:

Type I

$\underset{x\to a}{Lt} f(x) = f(a)$ $\quad \underset{x\to a}{Lt} \frac{f(x)}{g(x)} = \frac{f(a)}{g(a)}; if\ g(a) \neq 0$

Type II

$\underset{x\to a}{Lt} \frac{f(x)}{g(x)} \,\&\, g(a) = 0$, then cancel the common terms from numerator and denominator using algebraic treatments.

The reduced form would be: $\underset{x\to a}{Lt} \frac{f(x)}{g(x)} = \underset{x\to a}{Lt} \frac{p(x)}{q(x)} = \frac{p(a)}{q(a)}$

Type III

$\underset{x\to \infty}{Lt} \frac{f(x)}{g(x)}$, Divide numerator and denominator by the highest power of x, and then put 1/x = 0.

Type IV(Standard Limits)

- $\underset{x\to 0}{Lt} \frac{e^x - 1}{x} = 1 \quad \underset{x\to 0}{Lt} \frac{e^{mx} - 1}{x} = m \quad \underset{x\to 0}{Lt} \frac{e^{mx} - 1}{mx} = 1$
- $\underset{x\to 0}{Lt} \frac{a^x - 1}{x} = \log_e a \quad \underset{x\to 0}{Lt} \frac{a^{mx} - 1}{x} = m.\log_e a \quad \underset{x\to 0}{Lt} \frac{a^{mx} - 1}{mx} = \log_e a$
- $\underset{x\to 0}{Lt} \frac{\log(1+x)}{x} = 1 \quad \underset{x\to 0}{Lt} \frac{\log(1+mx)}{x} = m \quad \underset{x\to 0}{Lt} \frac{\log(1+mx)}{mx} = 1$
- $\underset{x\to a}{Lt} \frac{x^n - a^n}{x - a} = n.a^{n-1} \quad \underset{x\to a}{Lt} \frac{x^n - a^n}{x^m - a^m} = \frac{n.a^{n-1}}{m.a^{m-1}} = \frac{n}{m}.a^{n-m}$
- $\underset{x\to \infty}{Lt} \left(1 + \frac{1}{x}\right)^x = e \ \underset{x\to \infty}{Lt} \left(1 + \frac{a}{x}\right)^x = e^a$
- $\underset{x\to 0}{Lt} \left(1 + x\right)^{\frac{1}{x}} = e; \underset{x\to \infty}{Lt} \left(1 + x\right)^{\frac{a}{x}} = e^a; \underset{x\to 0}{Lt} \left(1 + ax\right)^{\frac{1}{x}} = e^a$

Type – I

1) $\lim\limits_{x \to 2} \frac{x^2 - x + 3}{x + 4}$

a) 3/6

b) 3/5

c) 1/5

d) 5/6

2) $\lim\limits_{x \to 1} \frac{4x^5 + 9x + 7}{3x^6 + x^3 + 1}$

a) 4

b) 5

c) 6

d) 2

Type II

3) $\lim\limits_{x \to 2} \frac{x^2 + x - 6}{x^2 - x - 2}$

a) 6/3

b) 5

c) 5/3

d) 3/5

4) $\lim\limits_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$

a) 1/6

b) 1/5

c) 1/9

d) 1/7

5) $\underset{x \to 0}{Lt} \frac{\sqrt{2+3x} - \sqrt{2-5x}}{4x}$

a) $\frac{1}{2\sqrt{2}}$

b) $\frac{1}{\sqrt{2}}$

c) $\sqrt{2}$

d) None of the above

6) $\underset{x \to 0}{Lt} \frac{1-\sqrt{1-x^2}}{x^2}$

a) $\frac{1}{3}$

b) $\frac{2}{5}$

c) $\frac{1}{2}$

d) None of the above

7) $\underset{k \to 0}{Lt} \frac{(2z+3k)^3 - 4k^2z}{2z(2z-k)^2}$

a) 1
b) 2
c) -1
d) -2

8) $\underset{x \to 0}{Lt} \frac{\sqrt{1+x} - \sqrt{1+x^2}}{\sqrt{1-x^2} - \sqrt{1-x}}$

a) 0
b) 2
c) 1
d) -1

9) $\underset{x\to 1}{\text{Lt}}\frac{x^2-3x+2}{x^3-4x+3}$

a) 0

b) 1

c) 2

d) 4

10) $\underset{x\to 1}{\text{Lt}}\frac{x^7-2x^5+1}{x^3-3x^2+2}$

a) 0

b) 1

c) 2

d) 4

Type – III – Limits, When the variable tends to Infinity

11) $\underset{x\to \infty}{\text{Lt}}\frac{x^2+3x+2}{x^3+x-4}$

a) 0

b) 1

c) 2

d) 4

12) $\underset{x\to \infty}{\text{Lt}}\frac{1+\sqrt{x}}{1-\sqrt{x}}$

a) 0

b) - 1

c) 1

d) 2

13) $\underset{h \to \infty}{Lt} \frac{3h + 2xh^2 + x^2h^3}{4 - 3xh - 2x^3h^3}$

a) $\frac{1}{x}$

b) $-\frac{1}{2x}$

c) $\frac{1}{3x}$

d) None of the above

14) $\underset{x \to \infty}{Lt} \frac{(1 + 2x^2)(3 - x^4)}{(1 + x^2)(5 + x^4)}$

a) -2

b) 2

c) 1

d) –1

15) $\underset{x \to \infty}{Lt} \frac{(1 + x)(1 - x^3)(2 + 3x + x^2)}{(5 - x^5)(6 + x)}$

a) 0

b) 1

c) 2

d) -1

16) $\underset{x \to \infty}{Lt} \frac{1 + 2 + 3 + ... + x}{x^2}$

a) $\frac{1}{2}$

b) $\frac{1}{3}$

c) $\frac{1}{4}$

d) None of the above

17) $\underset{x\to\infty}{Lt}\dfrac{1^2+2^2+3^2+...+x^2}{x^3}$

a) $\frac{1}{4}$

b) $\frac{1}{2}$

c) $\frac{1}{3}$

d) $\frac{2}{5}$

18) $\underset{x\to\infty}{Lt}\dfrac{1^3+2^3+3^3+...+x^3}{x^4}$

a) $\frac{1}{3}$

b) $-\frac{1}{4}$

c) $\frac{1}{4}$

d) None of the above

19) $\underset{n\to\infty}{Lt}\, 1+\frac{1}{2}+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^3+...t_n$

a) 1

b) 2

c) 3

d) 5

Type – IV - Definition

For each of the following functions (from Q No. 20 to25),evaluate the following limit:

$\underset{h\to 0}{Lt}\dfrac{f(x+h)-f(x)}{h}$

20) **f(x) =** $\frac{1}{x}$

a) $\frac{1}{x}$

b) $\frac{1}{x^2}$

c) $-\frac{1}{x^2}$

d) None of the above

21) f(x) = $\frac{1}{x^2}$

a) $\frac{1}{x^3}$

b) $\frac{2}{x^3}$

c) $-\frac{1}{x^3}$

d) $\frac{-2}{x^3}$

22) f(x) = $\sqrt{x}$

a) $\frac{1}{2x}$

b) $\frac{1}{2\sqrt{x}}$

c) $\frac{1}{\sqrt{x}}$

d) None of the above

23) f(x) = $\frac{1}{\sqrt{x}}$

a) $\frac{1}{2\sqrt{x}}$

b) $\frac{-1}{2\sqrt{x}}$

c) $\frac{-1}{2x\sqrt{x}}$

d) None of the above

24) f(x) = $ax^2 + bx + c$

a) ax

b) 2ax

c) 2ax + b

d) None of the above

25) f(x) = c (c = constant)

a) 0

b) 1

c) c

d) none of the above

Type – V – Standard Limits

26) $\underset{x\to 0}{Lt}\frac{7^{11x}-1}{x}$

a) $11.\log_7 e$

b) $7.\log_e 7$

c) $11.\log_e 7$

d) $11.\log_e 11$

27) $\underset{x\to 0}{Lt}\frac{e^{4x}-1}{x}$

a) 0

b) 4

c) 8

d) – 4

28) $\underset{x\to 0}{Lt}\frac{2^{-x}-1}{x}$

a) $-\log_e 2$

b) log 3

c) $\log_e 2$

d) none of the above

29) $\underset{x\to 0}{Lt}\frac{e^{-2x}-1}{x}$

a) 1

b) 2

c) -1

d) -2

30) $\underset{x \to a}{Lt} \dfrac{\sqrt[3]{x} - \sqrt[3]{a}}{x^{\frac{2}{3}} - a^{\frac{2}{3}}}$

a) $\frac{2}{3} a^{-\frac{1}{3}}$

b) $\frac{1}{3} a^{\frac{1}{3}}$

c) $\frac{1}{2} a^{-\frac{1}{3}}$

d) None of the above

31) $\underset{x \to 0}{Lt} \dfrac{e^{\alpha x} - e^{\beta x}}{x}$

a) $\alpha + \beta$

b) $\alpha . \beta$

c) $\alpha - \beta$

d) none of the above

32) $\underset{x \to 0}{Lt} \dfrac{3^x - 2^x}{x}$

a) $\log_e(3.2)$

b) $\log_e\left(\frac{2}{3}\right)$

c) $\log_e\left(\frac{3}{2}\right)$

d) none of the above

33) $\underset{x \to 0}{Lt} \dfrac{e^{\alpha x} + e^{\beta x} - 2}{x}$

a) $\alpha + \beta$

b) $\alpha . \beta$

c) $\alpha - \beta$

d) none of the above

34) $\underset{x\to 0}{\mathrm{Lt}}\dfrac{e^{5x}-e^{3x}-e^{2x}+1}{x^2}$

a) 3

b) 2

c) 6

d) -6

35) $\underset{x\to 0}{\mathrm{Lt}}\dfrac{6^x-3^x-2^x+1}{x^2}$

a) $\log_e 3 + \log_e 2$

b) $\log_e 3 - \log_e 2$

c) $\log_e 6$

d) $\log_e 3 \,.\, \log_e 2$

36) $\underset{h\to 0}{\mathrm{Lt}}\dfrac{e^{(x+h)^2}-e^{x^2}}{h}$

a) e^{x^2}

b) xe^{x^2}

c) $2xe^{x}$

d) $2\,xe^{x^2}$

37) $\underset{h\to 0}{\mathrm{Lt}}\dfrac{\log(x+h)-\log x}{h}$

a) x

b) 1

c) $\dfrac{1}{x}$

d) None of the above

38) $\underset{x \to e}{\text{Lt}} \frac{\log x - 1}{x - e}$

a) e

b) $\frac{1}{e}$

c) $\frac{2}{e}$

d) e^2

39) $\underset{x \to 2}{\text{Lt}} \frac{\log(2x-3)}{2(x-2)}$

e) 1

f) 2

g) -1

h) None of the above

40) $\underset{h \to 0}{\text{Lt}} \frac{(x+h)^n - x^n}{h}$

i) $n.\ x^{n-1}$

j) $n.a^{n-1}$

k) a^{n-1}

l) $n^2.a^{n-1}$

41). $\underset{x \to \infty}{\text{Lt}} \left(1 + \frac{a}{x}\right)^x$

a) e

b) e^a

c) e^3

d) e^{4a}

42) $\underset{x \to \infty}{\text{Lt}} \left(\frac{x+6}{x+1}\right)^{(x+6)}$

a. e
b. e^2
c. e^3
d. e^5

43) $\underset{x \to a}{\text{Lt}} \frac{xe^a - ae^x}{x-a}$

a. e^a
b. $e^a(1 +a)$
c. 1 + a
d. $e^a (1 -a)$

44) $\underset{x \to 0}{\text{Lt}} \{1+x\}^{1/4x}$

a. e
b. $e^{\frac{1}{2}}$
c. $e^{\frac{1}{4}}$
d. e^4

45) $\underset{x \to 0}{\text{Lt}} \{1+ax\}^{\frac{1}{x}}$

a. e
b. e^{2a}
c. e^{3a}
d. e^a

46) $\underset{x \to 2}{\text{Lt}} \frac{ax^2-b}{x-2} = 4$, find a & b.

a. 1, 2
b. 1, 3
c. 1, 1
d. 1, 4

47) $\underset{x \to 1}{\text{Lt}} \frac{ax^2+bx-2}{x-1} = 3$, find a & b.

a. 1, 1
b. 1, 2
c. 1, 3
d. 1, 4

CONCEPT OF CONTINUITY OF A FUNCTION

A function $f(x)$ is said to be Continuous at a particular point, x=a , if it satisfy the following conditions:

$$\lim_{x \to a^- \downarrow} f(x) \quad = \quad \lim_{x \to a^+ \downarrow} f(x) \quad = \quad \underset{\downarrow}{f}(a)$$

Left hand = Right hand =Functional

Limit (LHL) Limit (RHL) Value

Note1: Equality of RHL and LHL is treated as a condition for existence of limit i.e, limit of a function will exist if LHL=RHL

Note2: For Continuity, equality of the functional value at that point is also necessary.

Note3: For all Continuous functions, limit must exist, but existence of limit, is not a sufficient condition for continuity of a function.

Note4: Sum, difference , product and quotient of all continuous functions are always continuous.

Note5: All polynomials are continuous.

Note6: If a given function is of the form $\frac{f(x)}{g(x)}$, where both $f(x)$ and $g(x)$ are polynomials in x, it will be everywhere continuous except at the points at which it is undefined i.e; points of discontinuity of such functions are the points where $g(x)$=0.

Example: In each of the following cases, discuss continuity of the functions at x=5

i) $f(x)=\dfrac{x^2-25}{x-5}$

Solution: LHL= $\lim_{x\to 5^-}\dfrac{x^2-25}{x-5}=\lim_{x\to 5^-}\dfrac{2x}{1}=2\times 5=10$

RHL= $\lim_{x\to 5^+}\dfrac{x^2-25}{x-5}=\lim_{x\to 5^+}\dfrac{2x}{1}=2\times 5=10$

$f(5)=\dfrac{25-25}{5-5}=\dfrac{0}{0}(undifined)$

$\sin ce, LHL=RHL\neq f(5), f(x) is \quad discontinuous \quad at x=5, although$ the limit has existed.

ii) $f(x)=\dfrac{x^2-25}{x-5}, when \quad x\neq 5$

=10, when x=5

Solution: LHL=10=RHL taken from(i)

Given , $f(5)=10$ since, LHL=RHL= $f(5), f(x)$ is continuous at $x=5$

iii) $f(x)=\dfrac{x^2-25}{x-5}, when \quad x\neq 5$

=2, when $x=5$

Solution: LHL=RHL=10 taken from(ii)

Given , $f(5)=2$ since, LHL=RHL≠ $f(5), f(x)$ is discontinuous at $x=5$

Example 2: Find the points of discontinuity of the function, $f(x)=\dfrac{(x^2-3x+2)}{(x^2-5x+6)}$

Solution: The given function will be continuous at all points, except at the points at which it is undefined i.e the points at which its denominator is 0. $(x^2-5x+6)=0$

$\Rightarrow (x-2)(x-3)=0$

$\Rightarrow x=2,3$

Points of discontinuity are 2 and 3

WORKING CODES for Q. No. 1 to 18

Mark C : if function is continuous at the given point

Mark D : if function is discontinuous at the given point

Mark X : if nothing can be said about the continuity of the function at the given point

Mark Y : if function is neither continuous nor discontinuous at the given point

1) f(x) = $\frac{x^2-9}{x-3}$, Check continuity at x = 3

a) C

b) D

c) X

d) Y

2) f(x) = $\begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ 6, & \text{when } x = 3 \end{cases}$; Check continuity at x=3

a) X

b) Y

c) D

d) C

3) f(x) = $\begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ 1, & \text{when } x = 3 \end{cases}$; Check continuity at x = 3

a) C

b) X

c) D

d) Y

4) f(x) = $\begin{cases} x+1, x \geq 1 \\ 2x+1, x < 1 \end{cases}$, Check continuity x = 1

a) C

b) X

c) D

d) Y

5) f(x) = $\left.\begin{matrix} x^2, x > 2 \\ 4, x = 2 \\ 2x, x < 2 \end{matrix}\right\}$ Check continuity at x = 2

a) C

b) D

c) X

d) Y

6) f(x) = $\begin{cases} 4x+3, x \neq 4 \\ 3x+7, x = 4 \end{cases}$, Check continuity at x = 4

a) X

b) Y

c) C

d) D

7) f(x) = $\left.\begin{matrix} x^2, 0 < x < 1 \\ x, 1 \leq x < 2 \\ \frac{1}{4}x^3, 2 \leq x < 3 \end{matrix}\right\}$ Check continuity at x = 2

a) C

b) X

c) D

d) Y

8) f(x) = $\left.\begin{array}{l} 3+2x, -\frac{2}{3} \le x < 0 \\ 3-2x, 0 \le x < \frac{3}{2} \\ -3-2x,\ x \ \ge \frac{3}{2} \end{array}\right\}$ Check continuity at x = 0

a) D

b) Y

c) C

d) X

9) f(x) = $\left.\begin{array}{l} 3+2x, -\frac{2}{3} \le x < 0 \\ 3-2x, 0 \le x < \frac{3}{2} \\ -3-2x,\ x \ \ge \frac{3}{2} \end{array}\right\}$ Check continuity at x = $\frac{3}{2}$

a) C

b) D

c) X

d) Y

10) f(x) = $|x|$, Check continuity at x = 0

a) D

b) C

c) X

d) Y

11) $f(x) = \left.\begin{array}{l} \frac{|x-3|}{x-3}, \quad x \ne 3 \\ =1, \ \text{when} \ x = 3 \end{array}\right\}$, Check continuity at x = 3

a) X

b) Y

c) D

d) C

12) $f(x) = \left.\begin{array}{l} x + \dfrac{(x-2)}{|x-2|}, x \neq 2 \\ = \text{-1, when } x = 2 \end{array}\right\}$ Check continuity at x = 2

a) D

b) C

c) X

d) Y

13) f(x) = $|x-1|$ + 2x, Check continuity at x = 1

a) D

b) C

c) X

d) Y

(for Q. No. 14 and15)

Find the points of discontinuity of the following functions:

14) f(x) = $\dfrac{2x^2 - 6x + 5}{12x^2 + x - 20}$

a) $-\dfrac{4}{3}$

b) $\dfrac{5}{4}$

c) $\dfrac{4}{3}$

d) Both of a) and b) above

15) f(x) = $\dfrac{3x^2 - 4x}{x^3 + x^2 - x - 1}$

a) ± 1

b) 1

c) -1

d) None of the above

16. Given f(x) $\begin{cases} = x+1, & x \le 1 \\ = 4 - ax, & x > 1 \end{cases}$, for what value of a, will f(x) be continuous at x = 1?

   a) 2

   b) 1

   c) 3

   d) 4

17. Given f(x) $\begin{cases} = x+1, & x \le 1 \\ = 3 - ax^2, & x > 1 \end{cases}$, for what value of a, will f(x) be continuous at x = 1?

   a. 2

   b. 1

   c. 8

   d. 6

18. Given f(x) = $\frac{2x^2 - 8}{x - 2}$ is undefined at x = 2. What value must be assigned to f(2), if f(x) is to be continuous at x = 2 ?

   a. 6

   b. 2

   c. 8

   d. 1

19. Given f(x) = $\frac{1}{x}\left[\log(1+3x) - \log(1+2x)\right]$ is undefined at x = 0. What value must be assigned to f(0), if f(x) is to be continuous at x = 0 ?

   a. -1

   b. 0

   c. 1

   d. None of the above

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