

**CHAPTER 14**

**Integral Calculus**

**Fundamental Integrals**

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$	$\int \frac{dx}{x^n} = \frac{1}{(n-1).x^{n-1}} + C$	$\int \frac{dx}{\sqrt{x}} = 2\sqrt{x} + C$	$\int dx = x + C$
$\int \frac{dx}{x} = \log x  + C$	$\int e^x dx = e^x + C$	$\int e^{mx} dx = \frac{e^{mx}}{m} + C$	$\int a^x dx = \frac{a^x}{\log_e a} + C$
$\int a^{mx} dx = \frac{a^{mx}}{m \log_e a} + C$			

**Integration by Parts**

$$\int u.v dx = u.\int vdx - \int \left\{ \frac{du}{dx} \int vdx \right\} dx$$

**Standard Integrals**

- $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$ , Given  $(|x| > |a|)$
- $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$ , Given  $(|x| < |a|)$
- $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log \left| (x + \sqrt{x^2 \pm a^2}) \right| + C$
- $\int \sqrt{x^2 + a^2} dx = \frac{x\sqrt{x^2 + a^2}}{2} + \frac{a^2}{2} \log \left| x + \sqrt{x^2 + a^2} \right| + C$
- $\int \sqrt{x^2 - a^2} dx = \frac{x\sqrt{x^2 - a^2}}{2} - \frac{a^2}{2} \log \left| x + \sqrt{x^2 - a^2} \right| + C$

**Definite Integrals:- Important Properties**

$$\int_a^b f(x) dx = \int_a^b f(z) dz$$

$$\int_a^b f(x) dx = -\int_b^a f(x) dx$$

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx \quad (a < c < b)$$

$$\int_0^a f(x) dx = \int_0^a f(a - x) dx$$

$$\int_{-a}^{+a} f(x) dx = 0, \text{ if } f(x) \text{ is an odd function.}$$

$$\int_{-a}^{+a} f(x) dx = 2\int_0^a f(x) dx, \text{ if } f(x) \text{ is an even function.}$$

$$\int_a^b f(x) dx = \int_a^b f(a + b - x) dx$$

$$\int_a^b f(x) dx = \phi(b) - \phi(a)$$

$$\int_a^b x dx = \left. \frac{x^2}{2} \right|_a^b = \frac{b^2}{2} - \frac{a^2}{2} = \frac{b^2 - a^2}{2}$$

**Fundamental Integrals**

1.  $\int x^9 dx$

a)  $\frac{x^{10}}{10} + c$

b)  $\frac{x^{10}}{10}$

c)  $\frac{x^{11}}{11} + c$

d)  $\frac{x^8}{8} + c$

2.  $\int 5x^2 dx$

a)  $\frac{5}{3x^2} + c$

b)  $\frac{5x^3}{3} + c$

c)  $5x^3$

d) None of the above

3.  $\int \frac{1}{x^2} dx$

a)  $\frac{1}{x} + c$

b)  $\frac{1}{x^3} + c$

c)  $-\frac{1}{x} + c$

d) None of the above

4.  $\int 5^x dx$

a)  $\frac{5^x}{\log 5} + c$

b)  $\frac{5^x}{\log_5 e} + c$

c)  $5^x + c$

d)  $-5^x + c$

5.  $\int \left[ 5x^3 + 2x5^{-5} + 7x + \frac{1}{\sqrt{x}} + \frac{5}{x} \right] dx$

a)  $\frac{5x^4}{4} - \frac{1}{2x^4} + \frac{7x^2}{2} + 2\sqrt{x} + 5\log|x| + c$

b)  $\frac{5x^3}{3} + \frac{1}{2x^4} + \frac{7x^2}{2} + 2\sqrt{x} + 5\log|x| + c$

c)  $\frac{5x^4}{4} + \frac{1}{2x^4} + \frac{7x^2}{3} + 4\sqrt{x} + 5\log|x| + c$

d) None of the above

6.  $\int \left( x - \frac{1}{x} \right)^2 dx$

a)  $\frac{1}{3} \left( x - \frac{1}{x} \right)^3 + c$

b)  $\frac{x^3}{3} - 2x - \frac{1}{x} + c$

c)  $\frac{x^3}{3} + 2x + \frac{1}{x} + c$

d) None of the above

7.  $\int \left( \frac{x^4 + 1}{x^2} \right) + dx$

a)  $\frac{x^3}{3} - \frac{1}{x} + c$

b)  $\frac{1}{x} - \frac{x^3}{3} + c$

c)  $\frac{x^3}{3} + \frac{1}{x} + c$

d) None of the above

8.  $\int e^{-\log x} dx =$

- a)  $\log|x| + c$
- b)  $\log|x|$
- c)  $\log|x| + c$
- d) None of the above

9.  $\int e^{x \log a} e^x dx =$

- a)  $e^{x \log a} e^x + c$
- b)  $\frac{(ae)^x}{\log ae} + c$
- c)  $\frac{e^x}{1 + \log a} + c$
- d)  $\frac{(ae)^x}{\log a} + c$

10.  $\int e^{\frac{2}{3} \log \sqrt{x}} dx =$

- a)  $\frac{3}{4} x^{4/3} + c$
- b)  $\frac{3}{2} e^{\frac{2}{3} \log \sqrt{x}} + c$
- c)  $\frac{3}{5} e^{5/3} + c$
- d) None of the above

11.  $\int \frac{x^3 + 4x^2 - 3x - 2}{x+2} dx$

- a)  $\frac{1}{3}x^3 + x^2 - 7x + 12\log|x+2| + C$
- b)  $\frac{1}{3}x^3 + \frac{1}{2}x^2 - 7x + 12\log|x+2| + C$
- c)  $\frac{1}{3}x^3 + \frac{1}{2}x^2 - 7x + 2\log|x+2| + C$
- d) None of the above

12.  $\int (4-9x)^5$

- a)  $-\frac{(4-9x)^6}{54} + C$
- b)  $-\frac{(4-9x)^6}{54} + C$
- c)  $-\frac{(4-9x)^6}{54} + C$
- d)  $-\frac{(4-9x)^6}{54} + C$

13.  $\int \frac{dx}{(3-5x)}$

- a)  $\frac{\log(3-5x)}{5} + C$
- b)  $-\frac{\log(3-5x)}{5} + C$
- c)  $\frac{\log(3-5x)}{3} + C$
- d)  $-\frac{\log(3-5x)}{3} + C$

14.  $\int \sqrt{ax+b} \, dx$

a)  $\frac{2(ax+b)^3}{3a} + C$

b)  $\frac{2(ax+b)^2}{3a} + C$

c)  $-\frac{2(ax+b)^{2/3}}{3a} + C$

d)  $\frac{2(ax+b)^{3/2}}{3a} + C$

**Method of Substitution**

15.  $\int \frac{\log x}{x} \, dx$

a)  $\frac{\log x}{2} + C$

b)  $\frac{1}{2}(\log x)^2 + C$

c)  $\frac{1}{2}(\log x)^{2/3} + C$

d)  $\frac{3}{2}(\log x)^2 + C$

16.  $\int \frac{\log(x^2)}{x} \, dx$

a)  $\log x + C$

b)  $(\log x)^3 + C$

c)  $(\log x)^2 + C$

d) None of the above

17.  $\int \frac{(1 + \log x)^2}{x} dx$

- a)  $(x + \log x)^3 + C$
- b)  $3(x + \log x)^3 + C$
- c)  $\frac{1}{3} (1 + \log x)^3 + C$
- d) None of the above

18.  $\int \frac{1}{1 + e^{-x}} dx$

- a)  $\log|1 + e^{-x}| + c$
- b)  $\log(1 + e^x) + c$
- c)  $\log(e^x) + c$
- d) None

19.  $\int \frac{1}{x \log x} dx$

- a)  $\log|x| + c$
- b)  $\log|\log x| + c$
- c)  $\log(\log x + c)$
- d) None of the above

20.  $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$

- a)  $\log|e^x + e^{-x}| + c$
- b)  $\log|e^x - e^{-x}| + c$
- c)  $\log(e^x - e^{-x}) + c$
- d) None of the above



21.  $\int \frac{1}{x(\log x)^2} dx$

a)  $\frac{1}{\log x} + c$

b)  $\frac{-1}{\log x} + c$

c)  $\log x + c$

d) None of the above

22.  $\int e^{x^3} \cdot x^2 dx$

a)  $e^{x^3} + c$

b)  $\frac{1}{3} e^{x^3} + c$

c)  $\frac{x^6}{6} + c$

d) None of the above

23.  $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

a)  $e^{\sqrt{x}} + c$

b)  $\frac{1}{2} e^{\sqrt{x}} + c$

c)  $2 \cdot e^{\sqrt{x}} + c$

d) None of the above

24.  $\int \frac{1}{\sqrt{x}(1+\sqrt{x})} dx$

a)  $2 \log|1 + \sqrt{x}| + c$

b)  $\frac{1}{2} \log|1 + \sqrt{x}| + c$

c)  $\log|1 + \sqrt{x}| + c$

d) None of the above

25. Integrate w. r. t. x.  $1/[x \log x \log (\log x)]$

- a)  $\log [\log (\log x)] + c$
- b)  $\log (\log x) + c$
- c)  $\log x + c$
- d) None of the above

26.  $\int \frac{1}{(\sqrt{x} + x)} dx$

- a)  $\log|1 + \sqrt{x}| + c$
- b)  $2 \log|1 + \sqrt{x}| + c$
- c)  $\log|x + \sqrt{x}| + c$
- d) None of the above

27.  $\int x^x(1 + \log x) dx$  is equal to

- a)  $x^x \log x + c$
- b)  $e^{x^2} + c$
- c)  $\frac{x^2}{2} + c$
- d)  $x^x + c$

**Integration by Parts**

28.  $\int \log x \, dx$

a)  $\log|x| - x + c$

b)  $x \log|x| + c$

c)  $x \log \left| \frac{x}{e} \right| + c$

d) None of the above

29.  $\int x \log x \, dx$

a)  $\frac{x^2}{2} \log x - \frac{x^2}{4} + c$

b)  $\frac{x^2}{2} \log x + \frac{x^2}{4} + c$

c)  $\frac{x}{2} \log x - \frac{x^2}{4} + c$

d) None of the above

30.  $\int x^n \log x \, dx$

a)  $\frac{x^{n-1}}{(n+1)^2} [(n+1) \log x - 1] + c$

b)  $\frac{x^{n+1}}{(n+1)^2} [(n+1) \log x - 1] + c$

c)  $\frac{x^{n+1}}{n+1} [\log x - 1] + c$

d) None of the above

31.  $\int \frac{\log x}{x^2} dx$

- a)  $\frac{1}{x}(\log x + 1) + c$
- b)  $\frac{-1}{x}(\log x + 1) + c$
- c)  $(\log x - 1) + c$
- d) None of the above

32.  $\int xe^x dx$

- a)  $xe^x - e^x + c$
- b)  $e^x - xe^x + c$
- c)  $e^x + xe^x + c$
- d) None of the above

33.  $\int x^2 e^x dx$

- a)  $e^2(x^2 - 2x) + c$
- b)  $e^x(x^2 - 2x + 2) + c$
- c)  $e^x(x^2 + 2x - 2) + c$
- d) None of the above

34.  $\int x \cdot 2^x dx$

- a)  $\frac{2^x}{\log 2}(x + \log 2) + c$
- b)  $\frac{2^x}{\log 2}(\log 1/2) + c$
- c)  $\frac{2^x}{(\log 2)^2}(x \log 2 - 1) + c$
- d) None of the above

35.  $\int e^x \left\{ \frac{1}{x^2} - \frac{2}{x^3} \right\} dx$

a)  $e^x + C$

b)  $\frac{e^x}{x^2} + c$

c)  $-\frac{e^x}{x^2} + c$

d)  $\frac{e^x}{x} + c$

36.  $\int e^x \left\{ \frac{1+x \log x}{x} \right\} dx$

a)  $e^x + C$

b)  $e^x \log x + C$

c)  $x e^x + C$

d)  $e^{-x} \log x + C$

37.  $\int e^x \frac{x}{(1+x)^2} dx$

a)  $\frac{e^x}{x-1} + C$

b)  $\frac{e^x}{1+x} + C$

c)  $\frac{e^x}{1-x} + C$

d) None of the above

38.  $\int (x-1)e^x/x^2 dx$  is equal to

a)  $e^x/x + c$

b)  $e^{-x}/x + c$

c)  $-e^x/x + c$

d) None of the above

**Standard Integrals**

39.  $\int \frac{1}{9-x^2} dx$

a)  $\frac{1}{6} \log\left(\frac{3-x}{3+x}\right) + c$

b)  $\frac{1}{6} \log\left(\frac{3+x}{3-x}\right) + c$

c)  $\frac{1}{9} \log\left(\frac{3+x}{3-x}\right) + c$

d) None of the above

40.  $\int \frac{1}{\sqrt{x^2+9}} dx$

a)  $\log[x + \sqrt{x^2+9}] + c$

b)  $\frac{1}{2} \log[x + \sqrt{x^2+9}] + c$

c)  $\log[x - \sqrt{x^2+9}] + c$

d) None of the above

41.  $\int \frac{1}{\sqrt{x^2-25}} dx$

a)  $\log[x + \sqrt{x^2-25}] + c$

b)  $\frac{1}{2} \log[x + \sqrt{x^2-25}] + c$

c)  $\log[x - \sqrt{x^2-25}] + c$

d) None of the above

42.  $\int \sqrt{x^2+16} dx$

a)  $\frac{x}{2} \sqrt{x^2+16} + 8 \log[x + \sqrt{x^2+16}] + c$

b)  $\frac{x}{2} \sqrt{x^2+16} - 8 \log[x + \sqrt{x^2+16}] + c$

c)  $\frac{x}{2} \sqrt{x^2+16} + \frac{1}{8} \log[x + \sqrt{x^2+16}] + c$

d) None of the above

43.  $\int \sqrt{x^2 - 25} \, dx$

- a)  $\frac{x}{2}\sqrt{x^2 - 25} + \frac{25}{2}\log[x + \sqrt{x^2 - 25}]$   
b)  $\frac{x}{2}\sqrt{x^2 - 25} - \frac{25}{2}\log[x + \sqrt{x^2 - 25}] + c$   
c)  $\frac{x}{2}\sqrt{x^2 - 25} + \frac{2}{25}\log[x + \sqrt{x^2 - 25}] + c$   
d) None of the above

44.  $\int \frac{1}{9x^2 - 1} \, dx$

- a)  $\frac{1}{6} \left| \frac{3x-1}{3x+1} \right| \log + c$   
b)  $\frac{1}{6} \log \left| \frac{3x-1}{3x+1} \right| + c$   
c)  $\log \left| \frac{3x-1}{3x+1} \right| + c$   
d) None of the above

45.  $\int \sqrt{2x^2 + 5} \, dx$

- a)  $\frac{x\sqrt{2x^2+5}}{2} + \frac{5}{2\sqrt{2}} \log|\sqrt{2}x + \sqrt{2x^2+5}| + c$   
b)  $\frac{x\sqrt{2x^2+5}}{2} - \frac{5}{2\sqrt{2}} \log|\sqrt{2}x + \sqrt{2x^2+5}| + c$   
c)  $\frac{x\sqrt{2x^2+5}}{2} + \frac{5}{2\sqrt{2}} \log|\sqrt{2}x - \sqrt{2x^2+5}| + c$   
d) None of the above

**Partial Fractions**

46.  $\int \frac{1}{x(x+1)} dx$

a)  $\log \left| \frac{x}{(x+1)} \right| + c$

b)  $\log \left| \frac{x+1}{x} \right| + c$

c)  $\log \left| \frac{x^2}{1-x^2} \right| + c$

d) None of the above

47.  $\int \frac{1}{x(1-x)(1+x)} dx$

a)  $\frac{1}{2} \log \left| \frac{x^2}{(1+x^2)} \right| + c$

b)  $\frac{1}{2} \log \left| \frac{x^2}{(1-x^2)} \right| + c$

c)  $\log \left| \frac{x^2}{(1-x^2)} \right| + c$

d) None of the above

48.  $\int \frac{x dx}{(x-5)(x-6)}$

a)  $\log \left| \frac{(x-6)^6}{(x-5)^5} \right| + c$

b)  $\log \left| \frac{(x-5)^5}{(x-6)^6} \right| + c$

c)  $\log \left| \frac{(x-6)^5}{(x-5)^6} \right| + c$

d) None of the above



49.  $\int \frac{2x+3}{x^2+3x+2} dx$

- a)  $\log|x^2 + 3x + 2| + c$
- b)  $\log(x^2 + 3x + 2)^2 + c$
- c)  $\log(2x + 3) + c$
- d) None of the above

50.  $\int \frac{3x+2}{(x-2)(x-3)} dx$

- a)  $11\log|x-3| - 8\log|x-2| + c$
- b)  $11\log|x-2| - 8\log|x-3| + c$
- c)  $11\log|x-3| + 8\log|x-2| + c$
- d) None of the above

51.  $\int \frac{2x+1}{x(x+1)^2} dx$

- a)  $\log x - \log(x+1) - \frac{1}{x+1} + c$
- b)  $\log x + \log(x+1) + \frac{1}{x+1} + c$
- c)  $\log x - (x+1) + \frac{1}{x+1} + c$
- d) None of the above

52.  $\int \frac{2x}{(x^2+1)(x^2+2)} dx$

- a)  $\log|x^2 + 1| - \log|x^2 + 2| + c$
- b)  $\log|x^2 + 1| + \log|x^2 + 2| + c$
- c)  $\frac{1}{2}\log|x^2 + 1| + \log|x^2 + 2| + c$
- d)  $\frac{1}{3}\log|x^2 + 1| + \frac{2}{3}\log|x^2 + 2| + c$

**Definite Integrals**

53.  $\int_0^1 (2x+1) dx$

- a) 1
- b) 9
- c) 2
- d) 3

54.  $\int_1^2 (x^2 - 5x + 2) dx$

- a)  $\frac{19}{6}$
- b)  $-\frac{19}{6}$
- c)  $3\frac{5}{6}$
- d) None of the above

55.  $\int_1^5 \frac{dx}{7+2x}$

- a)  $\frac{1}{2} \log 3$
- b)  $\frac{1}{2} \log \left( \frac{17}{9} \right)$
- c) 1
- d) 0

56.  $\int_0^1 x^2 \cdot e^{3x} dx$

a)  $\frac{1}{27} [5e^3 - 2]$

b)  $\frac{11}{27} [5e^3 - 2]$

c)  $[5e^3 - 2]$

d)  $\frac{1}{27}$

57.  $\int_1^2 \frac{2x}{1+x^2} dx$

a)  $\log_e \frac{5}{2}$

b)  $\log 5 - \log_e 2 + 1$

c)  $\log_e \frac{2}{5}$

d) None of the above

58.  $\int_1^2 \frac{\log x}{x} dx$

a)  $\log 2$

b)  $2 \log 2$

c)  $3 \log 2$

d) None of these

59.  $\int_e^{e^2} \frac{1}{x(1+\log x)^2} dx$

a)  $\log \left| \frac{3}{2} \right|$

b)  $\frac{2}{3}$

c)  $\frac{3}{2}$

d) None of the above

$$60. \int_0^{\log 2} \frac{e^x}{e^{2x} + 3e^x + 2} dx$$

a)  $\log \left| \frac{3}{2} \right|$

b)  $\log \left| \frac{8}{9} \right|$

c)  $\log \left| \frac{9}{8} \right|$

d) None of the above

$$61. \int_0^1 x \cdot e^{2x} dx$$

a)  $\frac{1}{4}(e^2 - 1)$

b)  $\frac{1}{4}(e^2 + 1)$

c)  $\frac{1}{2}(e^2 - 1)$

d)  $\frac{1}{2}(e^2 + 1)$

$$62. \int_0^1 e^{\sqrt{x}} dx$$

a) 1

b) 2

c) 3

d) None of the above

63.  $\int_2^4 \frac{dx}{25-x^2}$

a)  $\frac{1}{10} \log\left(\frac{27}{7}\right)$

b)  $\frac{1}{10} \log\left(\frac{7}{27}\right)$

c)  $\frac{1}{11} \log\left(\frac{27}{7}\right)$

d)  $\frac{1}{11} \log\left(\frac{7}{27}\right)$

64.  $\int_0^{\log 3} \frac{e^x}{1+e^x} dx$

a)  $\log 3$

b)  $\log 2$

c) 1

d) None of the above

65.  $\int_{-2\sqrt{a^2-x^2}}^2 \frac{x dx}{\sqrt{a^2-x^2}}$

a) 0

b) a

c)  $a^2$

d) 1

66. The value of  $\int_2^3 f(5-x)dx - \int_2^3 f(x)dx$  is

a) 1

b) 0

c) -1

d) None of the above

67.  $\int_0^1 x(1-x)^n dx$

a)  $\frac{1}{(n+1)}$

b)  $\frac{1}{(n+2)}$

c)  $\frac{1}{(n+1)(n+2)}$

d)  $\frac{1}{n(n+2)}$

68.  $\int_2^5 \frac{\sqrt{x}}{\sqrt{7-x} + \sqrt{x}} dx$

a)  $\frac{5}{2}$

b)  $\frac{1}{2}$

c) 1

d) None of the above

69.  $\int_0^3 |x-2| dx$

a)  $\frac{1}{2}$

b)  $\frac{5}{2}$

c)  $-\frac{1}{2}$

d)  $-\frac{5}{2}$

70.  $\int_{-1}^1 |x| dx$

a) 1

b) 2

c) 0

d) None of the above

71.  $\int_0^2 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{2-x}} dx$

- a) 0
- b) 1
- c) 2
- d) 3

72.  $\int_0^2 \frac{x^2}{x^2 + (2-x)^2} dx$

- a) 2
- b) 0
- c) 1
- d)  $\frac{1}{2}$

**Applications of Integral Calculus**

73. Obtain the equation of the curve which passes through the point (3, 4) and the slope at the given point is in the ratio  $x : y$  with sign changed.

- a)  $x^2 - y^2 = 25$
- b)  $x^2 - y^2 = 16$
- c)  $x^2 + y^2 = 25$
- d)  $x^2 + y^2 = 9$

74. Given  $dy = (2x + 1) dx$  and  $y = 7$  when  $x = 1$ , find  $y$  when  $x = 3$ .

- a) 15
- b) 16
- c) 17
- d) 21

