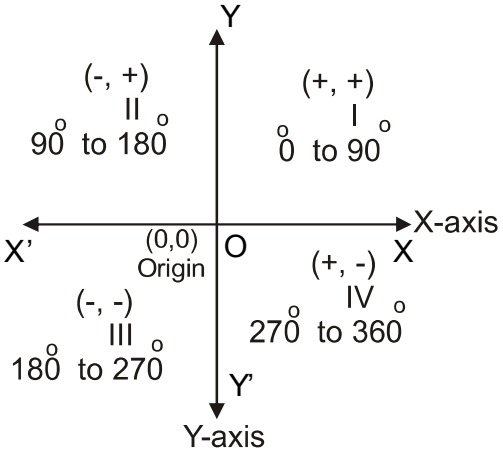
CHAPTER NO. 4 CO-ORDINATE GEOMETRY



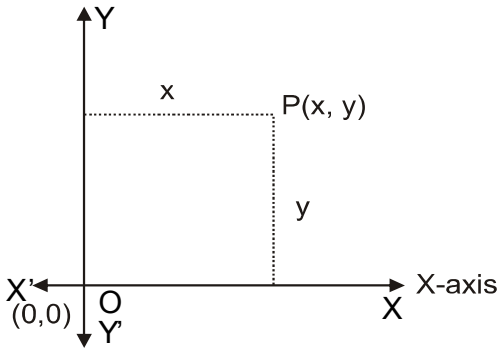
1. Distance formula and its applications

1.1 Distance between two points A and B where, A (x_1, y_1) and B (x_2, y_2)

AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Units

1.2 Distance of a point P(x, y) from the origin (0,0) is $\sqrt{(x^2 + y^2)}$ Units

1.3 Distance of a point P(x, y) from the X-axis is 'y' & from the Y-axis is 'x'



Application of distance formula to find out the nature of polygons:

2. Where there vertices A, B and C are given, A(x_1,y_1), B(x_2, y_2) and, (x_3, y_3) then

(a) If AB ≠ BC ≠ CA, then the triangle is a Scalene triangle

(b) If AB = BC = CA, then the triangle is Eqilateral triangle

(c) If any two sides are equal, then it is an Isosceles triangle and the unequal side is the base.

(d) If the sum of the squares of any two sides is equal to the square of the third side, then it is a Right angled triangle.

(e) If the sum of the squares of the equal sides of a triangle is equal to the square of the third side, then it is Right angled Isosceles triangle.

(f) When all sides are equal & the diagonals are also equal - [Square]

(g) Opposite sides are equal and diagonals are also equal - [Rectangle]

(h) Opposite sides are equal but diagonals are not equal - [Parallelogram]

(i) All sides are equal but diagonals are not equal.[Rhombus]

Note :

(1) In case of a parallelogram the diagonals bisect each other.

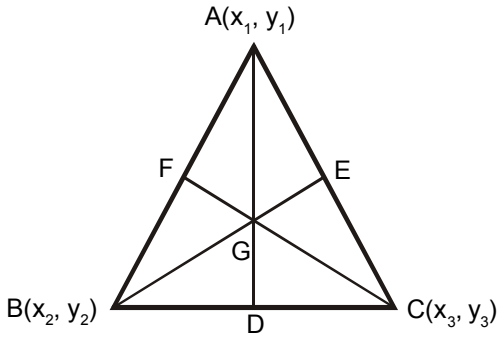
(2) In case of a rhombus the diagonals bisects each other.

(3) In all quadrilaterals (rhombus, parallelogram, square & rectangle diagonals bisect each other.

3. To find out the co-ordinate of the mid-point between $A(x_1, y_1)$ and $B(x_2, y_2)$

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

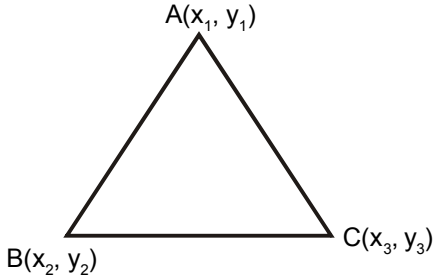
4. Centroid : Is the point of intersection of three medians where median is the side bisector from the vertex to the opposite side.

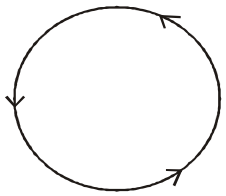


Here AD, BE, CF are the medians and G is the centroid whose co-ordinates are given by : $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

Note : Centroid divides the median in the ratio 2 : 1

5. To find the Area of Polygons :





Δ ABC = 1/2 $\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$ sq. units

6. **Condition for collinearity: If three points are collinear then the area of the triangle with the given vertices will be zero (0).**

7. **Area of Quadrilateral**

Area = 1/2 $\{(x_1y_2 - x_2y_1) + x_2y_3 - x_3y_3) + (x_3y_4 - x_4y_3) + (x_4y_1 - x_1y_4)\}$ sq. units

8.Section Formula:

If a point P(x, y) divides a line segment AB with co-ordinates A(x_1, y_1) & B{x_2, y_2) internally in ratio of m:n, then the co-ordinates of point P will be given by :

$$\left(\frac{mx_2 + nx_1}{m+n}, \quad \frac{my_2 + ny_1}{m+n}\right)$$ **Internal division**

If the division is external, then the co-ordinates of point P will be given by :

$$\left(\frac{mx_2 - nx_1}{m-n}, \quad \frac{my_2 - ny_1}{m-n}\right)$$ **External division**

9. CONCEPT OF SLOPE OF A STRAIGHT LINE

The slope or gradient of a straight line is the ratio of change in y co-ordinate to change in x co-ordinate & is denoted by :

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Change in y}}{\text{Change in x}} = \tan\theta = m$$

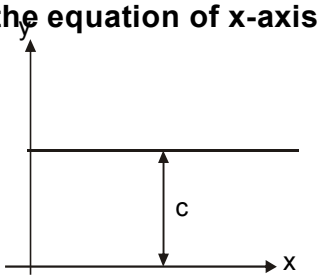
Note : Slope of a line can be + ve, - ve, 0 or ∞

STRAIGHT LINES

Different forms of Straight Line

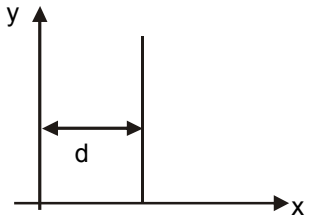
Case 1 : Equation of any line parallel to x-axis is of the form y = c

Note : When c = 0, the equation of x-axis is y = 0



Case 2 : Equation of a line parallel to y-axis of the form x = d

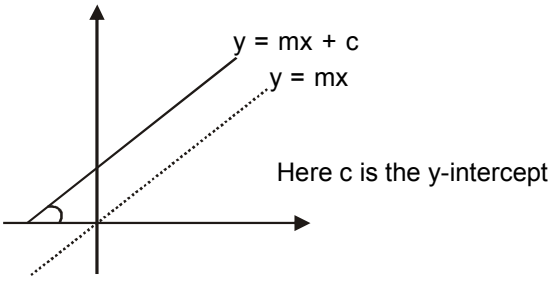
Note : When d = 0, equation of y-axis is x = 0



Case 3 : Slope intercept form :

The equation of the line having a slope 'm' & which cuts an intercept of 'c' units from the positive direction of Y-axis is given by y = mx + c

Note: When C= 0, y = mx will represents a line passing through the origin (0, 0)



Case 4 : Intercept Form :

If a straight line cuts of two intercepts of 'a' units & 'b' units from x-axis and y-axis respectively, then the equation of the straight line will be given by

$$\frac{x}{a} + \frac{y}{b} = 1$$

Note :

1. **When intercepts are equal in magnitude and are of same sign i.e. a = b, in such a case equation of the line will be**

   x/a + y/a = 1 OR x + y = a

2. **When intercepts are equal in magnitude but opposite in sign i.e. b = -a, in such a case the equation will be x/a - y/a = 1**

OR

x - y = a

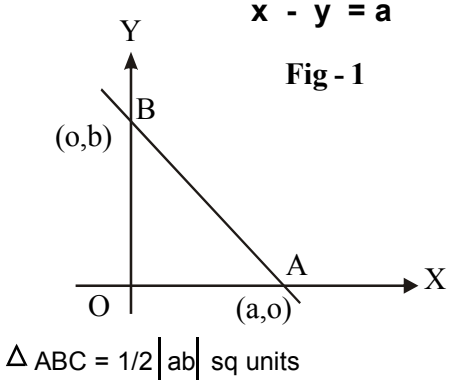


Fig - 1

Δ ABC = 1/2 |ab| sq units

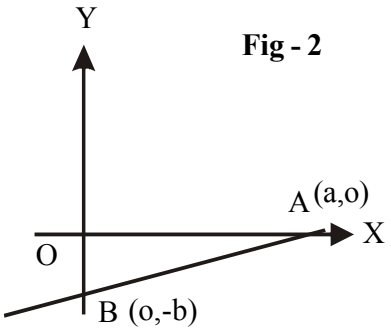


Fig - 2

Point- Slope form

Case - 5 : The equation of a line having a slope 'm' and which passes through a given point (x_1, y_1) will be given by $(y - y_1) = m (x - x_1)$

Point-point form

Case 6 : If a straight line passes through two given points $A(x_1, y_1)$ and $B(x_1, y_1)$, then the equation of such line will be given by B(x1 y1) put B(x2, y2).

Case 7: General equation of the straight line: It is the first degree equation in x & y. First degree means the power of the variable is one. Any equation of a straight line can be expressed in the general form ax + by + c = 0, then slope of which is given by m = -a/b

Case 8 : To find the point of intersection of the two lines, solve the two equations simultaneously.

Case 9 : Condition of concurrence of three straight lines
Three straight lines are said to be concurrent when the point of intersection of any two lines will satisfy the third line.

Case 10 : Parallel lines
Two lines are parallel when their slopes are equal

10.1 $a_1x + b_1y + c_1 = 0$ **and** $a_2x + b_2y + c_2 = 0$ **Now the given straight lines will be parallel if** $-a_1/b_1 = -a_2/b_2$

10.2 To find the equation of a line passing through a point (x_1, y_1) **and is parallel to the line** $ax + by + c = 0$ **will be** $ax + by + k = 0$. **On putting** (x_1, y_1) **in the new equation. Evaluate 'k' and with the value of 'k' form the required equation.**

Case 11 : Perpendicular lines
Straight lines are perpendicular to each other if the product of their slopes is (-1) i.e. $m_1 \times m_2 = -1$

11.1. The equation of a line to perpendicular to the given line $ax + by + c = 0$ **and passing through the point** $P(x_1, y_1)$**will be of the form,** $bx - ay + k = 0$
On putting (x_1, y_1) **in the new equation. Evaluate 'k' and with the value of 'k' form the required equation.**

Case12 : To find the perpendicular distance from a point $P(x_1, y_1)$ **from the straight line** $ax + by + c = 0$

$$\left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

12.1 When we have to find the perpendicular distance from the origin, we replace $P(x_1, y_1)$ **by (0, 0). The formula for finding out the perpendicular distance becomes.**

$$\left| \frac{c}{\sqrt{a^2 + b^2}} \right|$$

Q. 1. Find the perpendicular distance of the following cases:

(i) 4x+3y+5 = 0 from the point (1, 2)

(a) 3 (b) 5 (c) 2 (d) none

(ii) 4x+3y+5 = 0 from the point (-3, 1)

(a) -4/5 (b) 4/5 (c) 5/7 (d) 3/5

Case13 : To find whether points (x_1,y_1) & (x_2, y_2) will be on the same or opposite side of a given straight line ax + by + c = 0

Put the value of (x_1, y_1) and (x_2, y_2) in the given equation and observe the value, If the result is of the same sign (either both positive or both negative), it implies that the points are lying on the same side of the line, if the result are of opposite signs it will imply that the points will lie on the opposite sides of the given line.

Case 14 : To find the distance between two parallel lines $ax_1 + by_1 + c_1 = 0$ and $ax_1 + by_1 + c_2 = 0$

The distance

$$\text{'d'} = \left| \frac{c_1 \sim c_2}{\sqrt{a^2+b^2}} \right|$$

Special Case:

$\text{'d'} = \left| \frac{c_1 + c_2}{\sqrt{a^2+b^2}} \right|$ When the two lines will be on the opposite side of the origin.

$\text{'d'} = \left| \frac{c_1 - c_2}{\sqrt{a^2+b^2}} \right|$ When the two lines will be on the same side of the origin.

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NUMERICAL PROBLEMS

Q. 1. Find the distance of the points A(2, -3) and B(7,4)

(a) $\sqrt{74}$ units (b) $\sqrt{60}$ units (c) $\sqrt{75}$ units (d) $\sqrt{47}$ units

Q. 2. The square of the distance between the points A (2,5) and B(-3, 7) is given by

(a) 27 (b) 31 (c) 29 (d) 32

Q. 3. Let A (6, -1), B (1, 3) and C (x, 8) be three points such that AB = BC. Then the value of x is

(a) – 3 (b) 5

(c) Both of (a) and (b) above (d) None of the above

Q. 4. Points (6, 8), (3, 7), (-2, -2) and (1, -1) are joined to form a quadrilateral. What will be its structure?

(a) Rhombus (b) Parallelogram (c) Square (d) Rectangle

Q. 5. Points (4, -1), (6 , 0), (7, 2) and (5, 1) are joined to form a quadrilateral. What will be its structure?

(a) Rhombus (b) Parallelogram (c) Square (d) Rectangle

Q. 6. The triangle whose vertices are (0, 0), (2, 0) and (0, 3) is

(a) Acute angled (b) Isoceles (c) Equilateral (d) Right Angled

Q. 7. The extrimities of the diagonal of a parallelogram are the points (3,-4), and (-6, 5). If the third vertex is the point (-2, 1), then the co-ordinates of the fourth vertices are

(a) (1, 0) (b) (-1, 0) (c) (0, 1) (d) (0, -1)

Q. 8. Find out the mid-point between the point A(3, -1) and B(7, 5) ?

(a) (5, 2) (b) (5, 4) (c) (7, 3) (d) (2, 5)

Q. 9. Find out the mid-point between the point A (2, 3) and B (5, 2)

(a) (7/2, 5/2) (b) (9/2, 7/2) (c) (5/2, 3/2) (d) (7/2, 9/2)

Q. 10 Find the centroid of the three points A (5, 7), B (1, -3), C (-5, 1)

(a) (1/3, 5/3) (b) (7/3, 5/3) (c) (5/3, 2/3) (d) None of the above

Q. 11. Find the centroid of the three points A(3, 6), B(-5, 2), C (7, -6)

(a) (1/3, 5/3) (b) (5/3, 7/3) (c) (5/3, 2/3) (d) None of the above

Q. 12. If the vertices of a triangle have the co-ordinates (0, 0), (3, 0) and (0, 5). Find the area :

(a) 7.5 sq unit (b) 5.7 sq unit (c) 6.5 sq unit (d) None of the above

Q. 13. Find the area of the triangle whose vertices are (4, 4), (3, - 2) and (-3, 16).

(a) 27 (b) 36 (c) 49 (d) 26

Q. 14 If the points (1,1), (a,0) and (0, b) are collinear then what is the value of a + b?

(a) 0 (b) 1 (c) ab (d) None of the above

Q. 15. If (K, 2K), (2K, 3K) and (3, 1) are collinear, then K =

(a) 0 or –2 (b) 1 or 2 (c) 2 or -2 (d) 0 or 2

Q. 16. The coordinates of the angular points of a quadrilateral, taken in order, are (1, 1), (3, 4), (5, -2) and (4, -7). Find its area in sq. units.

(a) 41 (b) 21 (c) 20.5 (d) 21.5

Q. 17. Find the coordinates of point which divides the lines segmentnjoining(-1,3) and(4,7) i) internally ii) externally in the ratio 3:4.

(a) $\left(\frac{8}{7},-\frac{9}{7}\right) and (-16,33) respectively$ (b) $\left(\frac{6}{7},-\frac{9}{7}\right)$ & (2,4)

(c) $\left(-\frac{6}{7},\frac{9}{7}\right)$ & (1,3) (d)None

Q. 18.The point which divides the point of A (2, 3) & B (3, 7) internally in the ratio 2 : 3.

(a) (12/5, 23/5) (b) (23/5, 22/5) (c) (5, 6) (d) None of the above

Q. 19. The point which divides the join of (1, 2) and (3, 4) externally in the ratio 1 : 1

a) Lies in the IIIrd quadrant c) Lies in the Ist quadrant

b) Lies in the IInd quadrant d) Cannot be found

Q. 20. Find the slope from A (2, 3) & B (7, 5)

(a) 2/5 (b) 5/3 (c) 1/2 (d) None of the above

Q. 21. Find the slope from A (4, 6) & B (2,12)

(a) -3 (b) 3 (c) -5 (d) None of the above

(a) 3 (b) –3 (c) 1 (d) 0

Q. 22. Find the equation of the line parallel to x-axis which is at the distance of 10 units from x-axis

(a) y = 10 (b) x = 0 (c) y = 0 (d) None of the above

Q. 23. Find the equation of the line parallel to y-axis which is at the distance of 5 units from y-axis

(a) y = 10 (b) x = 5 (c) y = 5 (d) None of the above

Q. 24. Find the equation of the line whose slope is 1 and intercept is 3

(a) y = x + 3 (b) y = x + 5 (c) y = x + 6 (d) None of the above

Q. 25. Find the equation of the line whose slope is 1/2 and intercept is 2/3

(a) y = x/2 + 2/3 (b) y = x/3 + 2/5 (c) y = x + 6 (d) None of the above

Q. 26. Find the equation of the line whose slope is 2 and intercept is 0

(a) y = 2x (b) y = 3x + 5 (c) y = x + 6 (d) None of the above

Q. 27. Find the equation of the straight line which makes intercepts (-6, 4) on the axes of co-ordinates

x/-6 + y/4 = 1

(a) x + y = - 4 (b) 2x - 3y = 12 (c) 2x - 3y = 0 (d) 2x - 3y + 12 = 0

Q. 28. Find the equation of a line passing through (-4,1) and making equal intercepts on the co-ordinate axis

(a) x + 2y + 3 = 0 (b) x + y - 7 = 0 (c) x + 2y +12 = 0 (d) x + y + 3 = 0

Q. 29. Find the area of the region bounded by the line 3x + 4y = 7 with origin and the co-ordinate axes

(a) 49/24 (b) 24/39 (c) 2 (d) None of the above

Q. 30. Find the equation of the straight line that passes through the point (-5, -7) and has slope -0.5

(a) x/-19 + y/-19/2 = 1 (b) x/19 - y/19/2 = 1

(c) -x/19 - y/-19/2 = 1 (d) None of the above

Q. 31. Find the equation of the straight line passing through the points (1,2) and (0,5)

(a) 3x + y = -5 (b) x + 3y - 5 = 0

(c) 3x + y - 5 = 0 (d) None of the above

Q. 32. Find the equation of the straight line joining the points (1,1) and (-4,3)

(a) $y = \frac{2}{3}x - \frac{7}{5}$ (b) $y = \frac{2}{5}x + \frac{7}{5}$

(c) $y = -\frac{2}{3}x - \frac{7}{5}$ (d) $y = -\frac{2}{5}x + \frac{7}{5}$

Q. 33. Find the point of intersection of the straight lines 3x + 4y - 11 = 0 and x - 5y + 9 = 0

(a) (2, 1) (b) (1, 1) (c) (1, 2) (d) (-1, -2)

Q. 34. Find the point of intersection of the straight lines 4x + 5y – 11 = 0 and x – 5y + 9 = 0

(a) (2, 1) (b) (1, 1) (c) (1, 2) (d) None of these

Q. 35. Prove that 3x + y = 14
x - 2y = 0
5x - 8y = 4 are concurrent

Q. 36. Find the value of k so that the lines 2x - 3y + k = 0, 3x - 4y - 13 = 0 and 8x - 11y - 33 = 0 are concurrent

(a) 7 (b) -7 (c) 5 (d) -5

Q. 37. Find the equation of a line which is parallel to 2x + 3y + 7 = 0 and is passing through a point (1, 2)

(a) 2x + 3y -8 = 0 (b) 3x + 4y + 7 = 0

(c) -2x + 3y - 5 = 0 (d) None of the these

Q. 38. Find the equation of a line which is parallel to 3x - 4y + 15 = 0 and is passing through a point (3,1)

(a) 2x + 3y -8 = 0 (b) 3x + 4y + 7 = 0

(c) 3x - 4y - 5 = 0 (d) None of the these

Q. 39. Find the equation of the line passing through a point (1, 2) and which perpendicular to the line 2x - 3y + 7 = 0

(a) 3x + 2y - 7 = 0 (b) 3x - 4y + 7 = 0

(c) 5x - 2y - 5 = 0 (d) None of these

Q. 40. Find the equation of the straight line which passes through the point (-1, 3) and perpendicular to the straight line 4x + 3y + 1 = 0

(a) 3x - 4y = -15 (b) 3x - 4y = 15

(c) 3x - 4y = -51 (d) None of these

Q. 41. The equation of the straight line passing through the point (2,-4) and perpendicular to the line 8x - 4y + 7 = 0

(a) x + 2y + 6 = 0 (b) x - 2y + 6 = 0

(c) 2x + y + 6 = 0 (d) 2x - y + 6 = 0

Q. 42. The coordinates of the foot of the perpendicular from the point (2, 4) on the line x + y = 1 are:

(a) $\left(\frac{1}{2}, \frac{3}{2}\right)$ (b) $\left(-\frac{1}{2}, \frac{3}{2}\right)$ (c) $\left(\frac{4}{3}, \frac{1}{2}\right)$ (d) None of the above

Q. 43. Find the perpendicular distance of the following cases:

(i) 4x+3y+5 = 0 from the point (1, 2)

(a) 3 (b) 5 (c) 2 (d) none

(ii) 4x+3y+5 = 0 from the point (-3, 1)

(a) -4/5 (b) 4/5 (c) 5/7 (d) 3/5

Q. 44. Find the position of the points (3, 2) & (3, -1) with respect to the line x - 2y + 2 = 0

Q. 45. Find the position of the points (0, 0) & (2, 3) with repect to the line 7x - 24y + 8 = 0

MISCELLANEOUS PROBLEMS

Q. 46. The sides of a triangle lie on the lines y = x, y = 2x and y = 3x + 4. Find the co-ordinate of of its centroid.

(a) (-2, -10/3) (b) (-2, 10/3) (c) (2, -10/3) (d) None

Q. 47. The equation of the perpendicular bisector of the line segment joining the points (1,4) and (3, 6)

(a) x - y - 7 = 0 (b) x + y - 7 = 0

(c) x + y + 7 = 0 (d) none

Q. 48. If the lines ax + 12y + 1 = 0, bx + 13y + 1 = 0 and cx + 14y + 1 = 0 are con-current, then a, b, c are in :

(a) AP (b) GP (c) HP (d) None of the above

Q. 49. The area of the quadrilateral whose vertices are (-3, 2), (7, -6), (-5, -4) and (5, 4) is

(a) 80 sq. units (b) 70 sq. units (c) 90 sq. units (d) 16 sq. units

Q. 50. The ratio which the line y - x + 2 = 0 divides the line joining (3, -1) and (8, 9) is

a) 2 : 3 b) 3 : 2 c) -2 : 3 d) -3 : 2