||श्री||

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Chapter → 13
Theoretical Distribution

Probability distribution [ऐसा table जिसमे Probability होती है वह है Probability distⁿ]

↓

Table

Discrete distribution

X	P
1	0.2
2	0.3
3	0.4
4	0.1

(a) Binomial distⁿ
(b) Poisson distⁿ

↓

formula / Probabilities function

↓

Probability Mass function [PMF]

↓

$\Sigma P = 1$

Continuous distribution

Class	P
0-10	0.3
10-20	0.4
20-30	0.3

(a) Normal distⁿ
~~(b)~~ T-distⁿ
~~(c)~~ Chi-square distⁿ
~~(d)~~ F-distⁿ

↓

formula / Probabilities function

↓

Probability Density function PDF

↓

$\int P = 1$

$x - 0, 1, 2, 3 \dots n$

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Binomial distribution :- $x \sim B(n, P)$

- Variable should be discrete
- Problem should be of success and failure
- n & P are moderate $[n < 100]$

$\boxed{x \sim B(n, P)}$ x is a varaible which follows Binomial parameter n & p.

formula | Probability function →

$$F(x) = nC_x \cdot p^x \cdot q^{n-x}$$

$x = 0, 1, 2, 3 \dots n$

- n = no. of trials
- p = Probability of Success

q = Probabity of failure

$[1 - P]$

eg → 25 coins are tossed what is probability of 4 Heads.

x = no. of heads
$= 0, 1, 2, 3 \dots 25$
$n = 25$

P[4 heads]
p = Head = success
q = Tail = failure

Properties of Binomial :-

1. • It is biparametric distribution [n & P]
2. • $\Sigma P = 1$
3. • Mean = np
4. • Variance = npq
   S.D = $\sqrt{npq}$
5. • Maximum variance ⇒ $\left[p = q = \frac{1}{2} \right]$ → Symmetric distribution
   ↳ $\frac{n}{4}$
6. • Mode = $(n+1)p$

Unimodal ↓ non-integer

Bimodal ↓ Integer — $(n+1)p$, $(n+1)p - 1$

$x = 0, 1, 2, 3 \ldots\ldots n$

Poison distribution :→ $x \sim P(m)$

- variable should be discrete
- Problem should be of success & failure
- n is very large & p is very small $[n > 100]$

$m = np$

Formula / Probability function :→

$$f(x) = \frac{e^{-m} \cdot m^{x}}{x!}$$

$e = 2.7182$

x = will be given

m

Properties of Poision :→

1. It is uniparametric distribution (m)
2. $\Sigma P = 1$
3. Mean $= m$
4. Variance $= m$

   S.D $= \sqrt{m}$
5. mode $= m$

non integer → Unimodal

eg → 8.3, 7.5

8.3 → 8

7.5 → 7

integer → Bimodal

e.g → 7, 15

7 → 7, 6

15 → 15, 14

$x = 0, 1, 2, 3 \ldots \infty$ μ → meu

x = can be -ve also

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Normal distribution :- $x \sim N[\mu, \sigma^2]$

- Continuous distribution

formula/ Probability function :→

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

Properties of Normal :→

1. It is biparmateric distⁿ $[\mu, \sigma^2]$
2. Mean = Median = Mode = μ
   [symmetric distribution]
3. S.D = σ
   Variance = σ^2
4. Mean Deviation
   MD = 0.8σ
5. Quartile Deviation
   $Q_1 = \mu - 0.675\sigma$
   $Q_2 = \mu$
   $Q_3 = \mu + 0.675\sigma$
   $QD = 0.675\sigma$
6. Point of inflexion
   $\mu - \sigma$
   $\mu + \sigma$

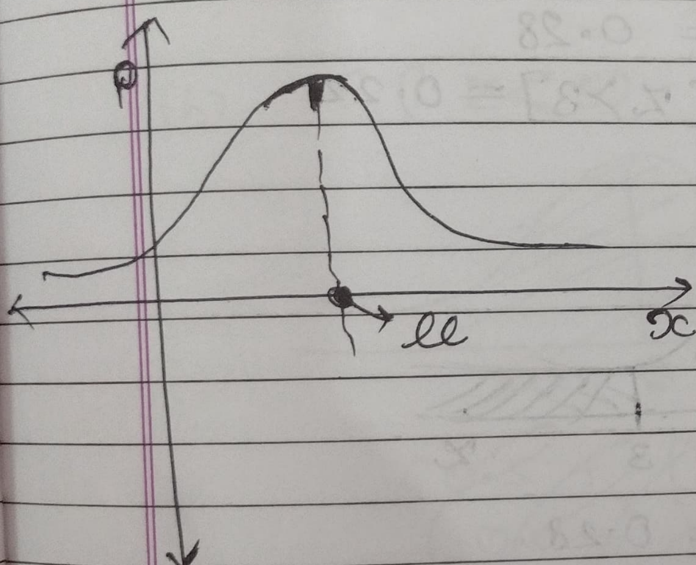
Standard Normal Distribution :→ $Z \sim N(0,1)$

$\mu = 0$

$\sigma = 1$

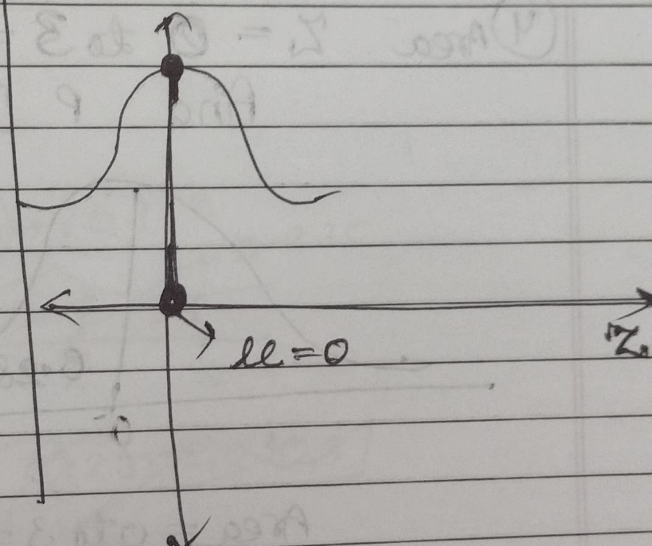
Normal Distribution

x



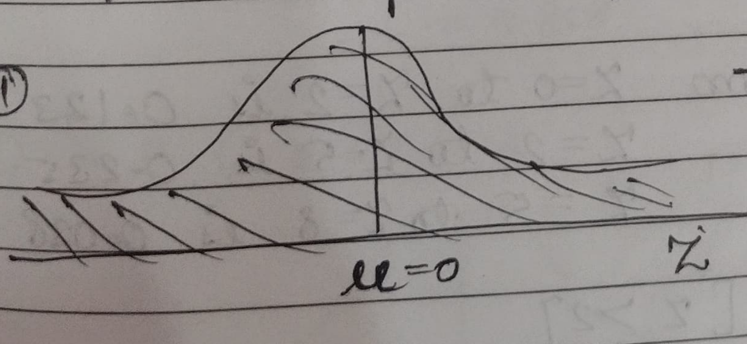
Standard Distribution

$z = \frac{x-\mu}{\sigma}$



How to solve problems of probability →

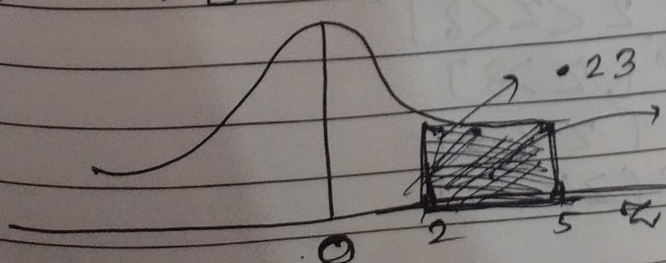
① $-\infty \le Z \le \infty$



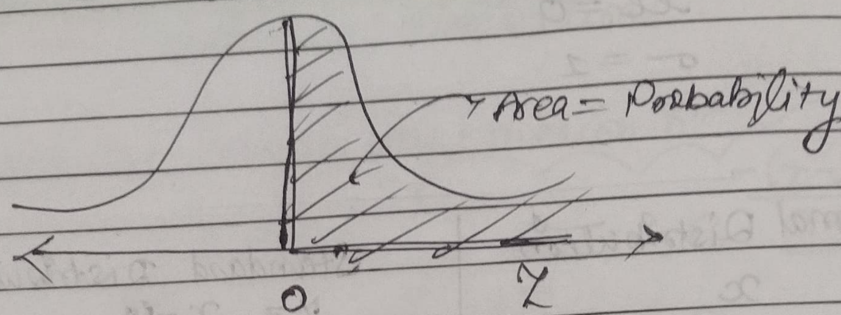
Total Area

$z = 1$

② $P[2 < Z < 5]$

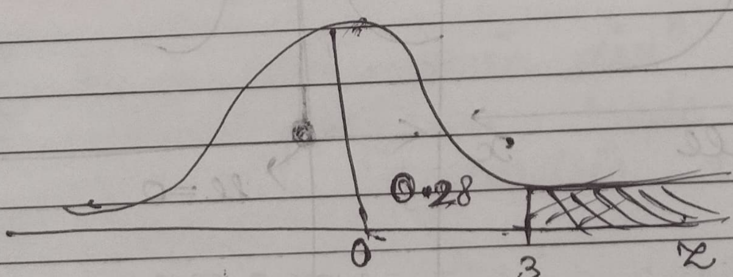


(3) $P[Z > 0] = \underline{0.5}$



(4) Area $Z = 0$ to $3 = 0.28$

find $P[Z > 3] = 0.22$



Area = 0 to 3 = 0.28

$0.5 - 0.28 = 0.22$

(5) Area from $Z = 0$ to $Z = 2$ is 0.123

$Z = 2$ to $Z = 5$ is 0.235

$Z = 5$ to $Z = 8$ is 0.038

find

(a) $P[Z > 2]$

(b) $P[Z < 5]$

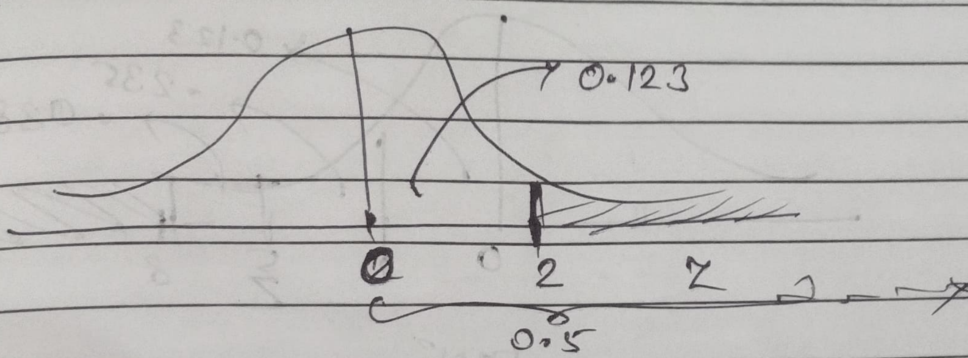
(c) $P[2 < Z < 8]$

(d) $P[Z > 8]$

(e) $P[Z < -5]$

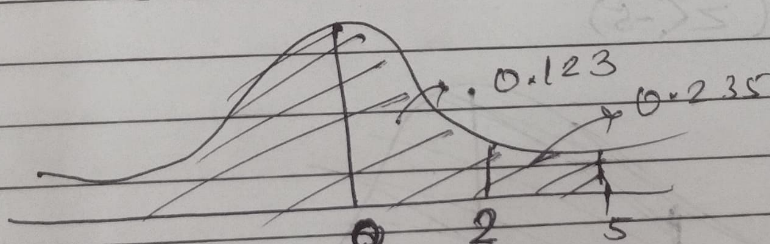
(f) $P(Z > -2)$

(a) $P(Z > 2)$



$0.5 - 0.123 = 0.377$

(b) $P(Z < 5)$



~~$(0.5 + [0.235 - 0.123]$~~

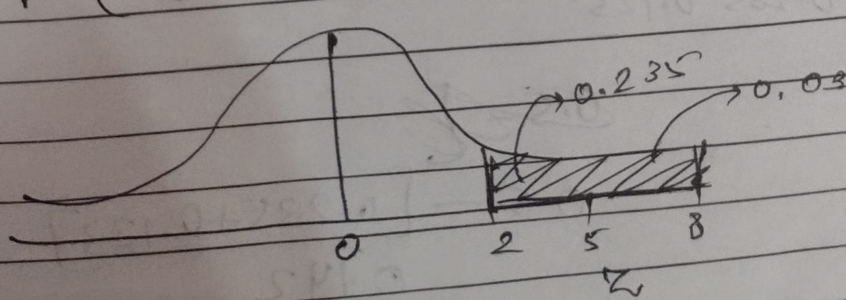
~~$0.5 + 0.112$~~

~~$.612$~~

$0.5 + 0.123 + 0.235$

$= 0.858$

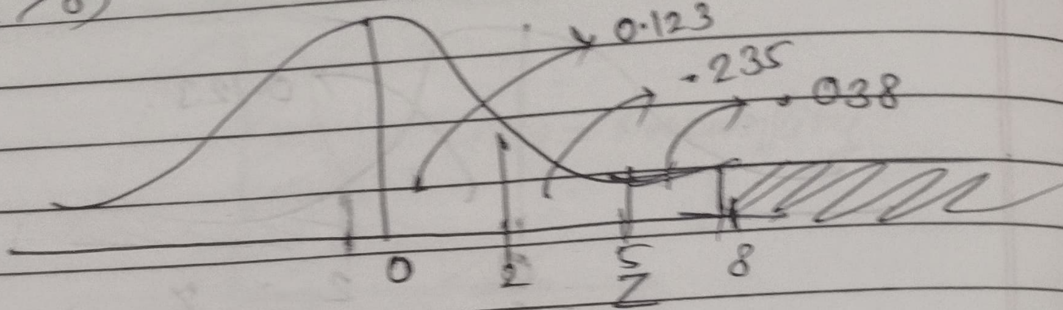
(d) $P(2 < Z < 8)$



$0.235 + 0.038$

0.273

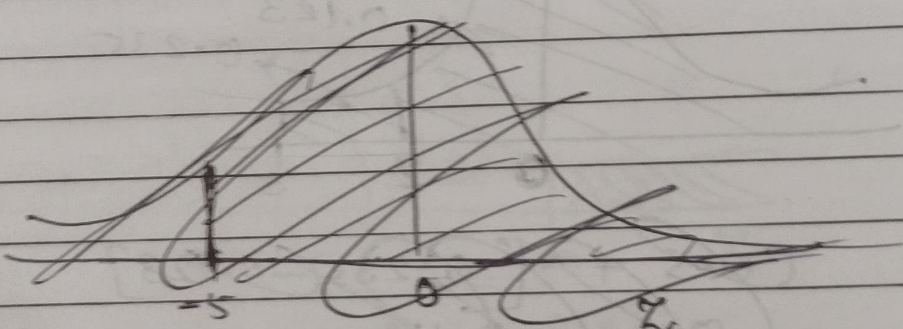
(c) $(z > 8)$

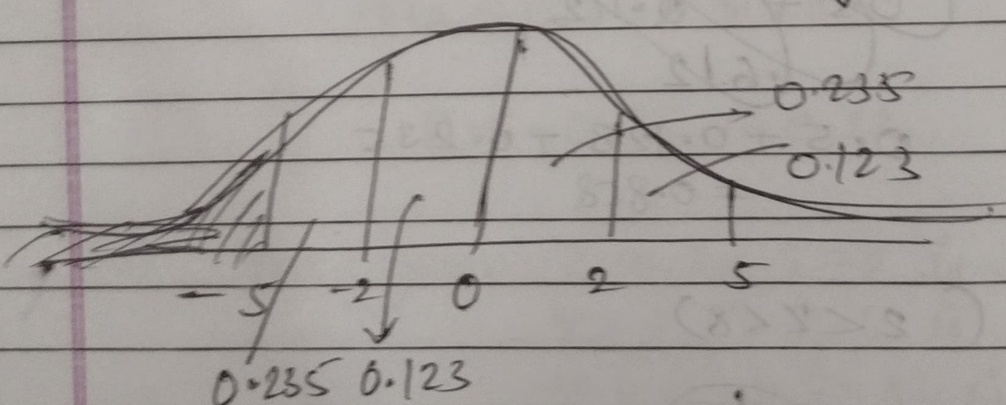


$1 - [0.123 + 0.235 + 0.038]$

~~0.604~~ 0.104

(e) $P(z < -5)$

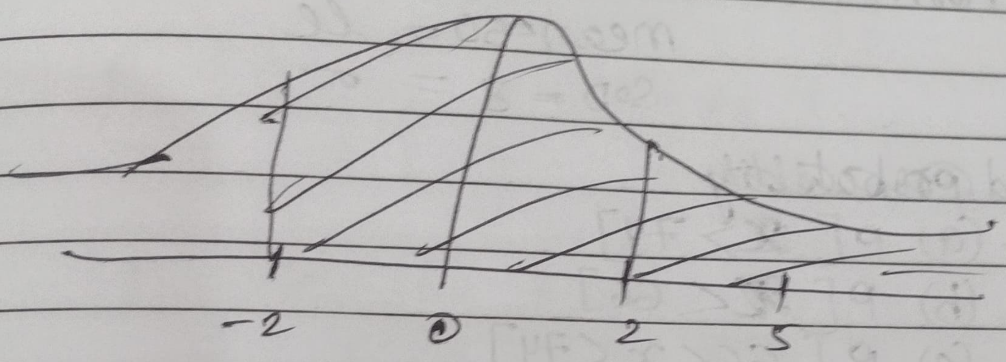




$0.5 - [0.235 + 0.123]$

0.142

(f) $P(Z > -2)$



$\cancel{0.5 - 0.123}$

$0.5 + 0.123$

0.623

Steps to find Probability in normal distribution

① Convert x into z using the formula

$$z = \frac{x - \mu}{\sigma}$$

② Draw graph of z

③ Find area or probability using data given in question or standard normal table

Q For normal distribution :→

$$mean = 50 = \mu$$
$$S.D = 8 = \sigma$$

find ~~probability~~

(a) $P[x > 74]$
(b) $P[x < 66]$
(c) $P[66 < x < 74]$

Area from $z = 0$ to $z = 2 \rightarrow 0.125$
$z = 2$ to $z = 3 \rightarrow 0.138$

(a) $P[x > 74]$

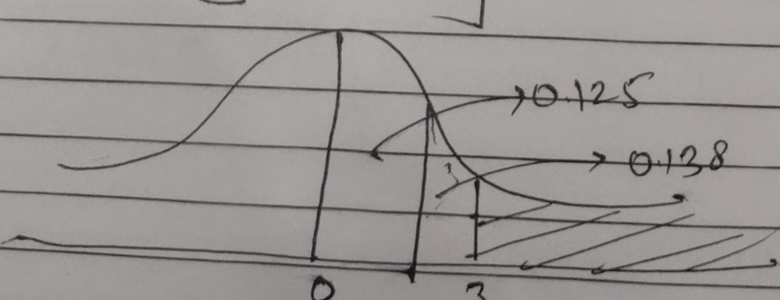
$$z = \frac{x - \mu}{\sigma}$$

$$P\left[\frac{x-\mu}{\sigma} > \frac{74-\mu}{\sigma}\right]$$

$$P\left[\frac{x-\mu}{\sigma} > \frac{74-50}{8}\right]$$

24

$$P[z > 3]$$



$0.5 - 0.125 - 0.138$

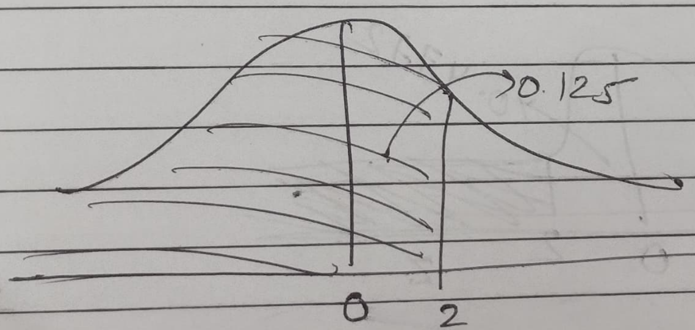
0.237

(b) $P[x < 66]$

$$P\left[\frac{x-\mu}{\sigma} < \frac{66-\mu}{\sigma}\right]$$

$$P\left[\frac{x-\mu}{\sigma} < \frac{66-50}{8}\right]$$

$$P[Z < 2]$$



$0.5 + 0.125$

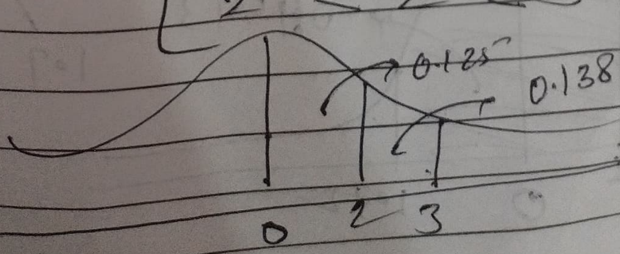
0.625

(c) $P[66 < x < 74]$

$$P\left[\frac{66-\mu}{\sigma} < \frac{x-\mu}{\sigma} < \frac{74-\mu}{\sigma}\right]$$

$$\frac{66-50}{8} < Z < \frac{74-50}{8}$$

$$[2 < Z < 3]$$



$\underline{\underline{0.138}}$

(2) $\mu = 1000 \rightarrow$ mean

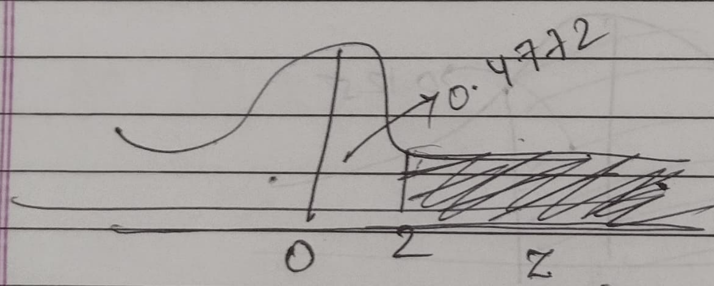
$\sigma = 25 \rightarrow$ S.D

(a) $P[\text{income} > 1050]$

$[x > 1050]$

$$P\left[\frac{x-\mu}{\sigma} > \frac{1050-\mu}{\sigma}\right]$$

$P[Z > 2]$



2.00

2.0   0.00

.4772

$.5 - 0.4772$

0.0228

(b) $P[\text{income} < 1043]$

$P[x < 1043]$

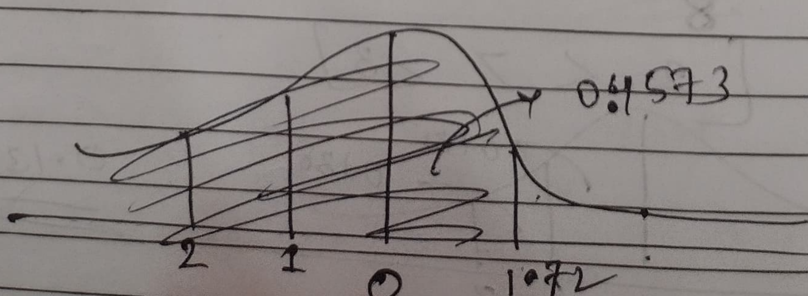
$$P\left[\frac{x-\mu}{\sigma} < \frac{1043-\mu}{\sigma}\right]$$

$P[Z < 1.72]$

1.72

1.7   0.02

0.4573



$0.5 + 0.4573$
$= 0.9573$

\# $\phi(7)$

$P[Z >)$

$P(Z < 3)$
$\phi(3)$

ϕ_1
$P(Z < 1)$