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Statistics = 40 marks.

Chapter → 10
- A. Central Tendency
- B. Dispersion

⇒ Central Tendency :→

Central Tendency
- Arithmetic Mean [A.M.]
  - → Combined Mean
  - → Weighted Mean
- Mode
- Median
- Harmonic Mean [H.M.]
- Geometric Mean [G.M.]

Central Tendency
↓
Averages
- Mathematical Avgs. (↓ formula)
  - A.M. (X)
  - G.M.
  - H.M.
- Positional Avg / Graphical Avg (↓ Graph)
  - Median (Me) → ogive graph
  - Mode (Mo) → Histogram
  - Quartile, Decile & Percentile.

Central Tendency :→ Single no. which represent whole data called Central Tendency.

Discrete → Single
Continuous → Group

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① Arithmetic Mean [A.M.]

- **Discrete data without frequency**

$$\bar{X} = \frac{\Sigma X}{N}$$

n = no. of observation

- **Discrete data with frequency**

$$\bar{X} = \frac{\Sigma fx}{\Sigma f}$$

- **Continuous data**

$$\bar{X} = \frac{\Sigma fx}{\Sigma f}$$

[x = mid point] → $\frac{\text{upper limit} + \text{lower limit}}{2}$

② Median → [ ] ⟶ Middle Most Value

- **Discrete data without frequency**
  (i) Arrange data in ascending order.
  (ii) Median = $\left[\frac{N+1}{2}\right]^{th}$ value

n = no. of observation

- **Discrete data with frequency**
  Step1 :- find less than c.f.
  Step2 :- find $\left[\frac{N+1}{2}\right]$ [N = Σf]

③ M

Boundary → lowerlimit −0.5 Upperlimit +0.5
[Inclusive Series]

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Step 3 :→ Check that $C \geq \frac{N+1}{2}$

Step 4 :→ C.f. ⟶ X ⟶ Median

- **Continuous Data**

Step 1 :→ Find less than C.f.

Step 2 :→ Find $\frac{n}{2}$ $[\Sigma F = n]$

Step 3 :→ Check that C.f. $\geq n/2$

Step 4 :→ C.f. ⟶ Class ⟶ Median class

$$\text{Median} = l_1 + \left[\frac{n/2 - C.f.}{f}\right] \times h$$

l = lower class Boundary of median class

h = class width = UCB − LCB

f = frequency of Median class

C.f. = Cumulative frequency of Previous class.

③ **Mode** ⟶ ⟶ Most recurring.

→ Value having maximum repetition or maximum frequency.

- **Discrete data without Frequency**

maximum repetition → mode

- **Discrete data with frequency**

maximum frequency → mode. — x

- **Continuous data**
  - Maximum frequency ⟶ 0 ⟶ Modal class

$$\text{Mode} = l_1 + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

l_1 = LCB of modal class
h = UCB − LCB → class width
f_0 = previous class frequency
f_1 = Maximum frequency
f_2 = next class frequency

(4) Harmonic Mean [H.M.]

- **Discrete data without frequency**

$$H.M_0 = \frac{N}{\Sigma 1/x}$$

~~x = midpoint~~

- **Discrete data with frequency**

$$H.M_0 = \frac{\Sigma f}{\Sigma f/x}$$

- **Continuous data**

$$H.M_0 = \frac{\Sigma f}{\Sigma f/x}$$

[x = midpoint]

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⑤ Geometric Mean [G.M.]

- **Discrete data without frequency**

$$G.M. = [x_1 \cdot x_2 \cdot x_3 \cdots x_N]^{1/N}$$

- **Discrete data with frequency**

$$G.M. = [x_1^{f_1} \cdot x_2^{f_2} \cdot x_3^{f_3} \cdots]^{1/\Sigma f}$$

- **Continuous data**

$$G.M. = [x_1^{f_1} \cdot x_2^{f_2} \cdot x_3^{f_3} \cdots]^{1/\Sigma f}$$

[x = midpoint]

* Best/most used ⟶ A.M
* Most Popular ⟶ Mode
* Easy to Calculate ⟶ Mode
* Hard to Calculate ⟶ G.M
* Based on all Observation ⟶ AM, GM, HM
* Have Mathametical Property [combined kaformula] ⟶ AM, GM, HM
* Affected by Sampling fluctuating ⟶ AM
* Open end Class ⟶ Median (Best), Mode
* Can't be defined (sometime) ⟶ Mode.
* rigidly defined → formula होना
* extreme Observation → 1st & last no. se.
* AM ≥ GM ≥ HM , $GM^2 = AM \times H$
* Used to find out Avg Interest rates → GM & HM

deviation = difference.

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Properties of Mean, Median and Mode.

① If all observations are same then mean, median and mode are also same.

7, 7, 7, 7, 7

Mean = Median = Mode = 7.

② Relation Between mean, median and mode

Case i :→ For Symmetric data

Mean = Median = Mode

Case ii :→ for Asymmetric data

Mode = 3 Median − 2 Mean

③ Combined Arithmetic Mean

① $N_1 = 70$ $\bar{X}_1 = 60$ ② $N_2 = 30$ $\bar{X}_2 = 70$ (1,2) $N_1 + N_2 = 100$ $\bar{X}_{12}$

$$\bar{X}_{12} = \frac{N_1\bar{X}_1 + N_2\bar{X}_2}{N_1 + N_2}$$

④ Change of scale and change of origin.

Mean, Median and Mode are affected By Both Change of scale $(\times, \div)$ and Change of Origin $(+, -)$

$$y = a + bx$$

$\bar{y} = a + b\bar{x}$

Median $y = a + b$ median

Mode $y = a + b$ Mode.

⑤ Sum of deviation of all observation about arithmetic mean is zero (0).

X	$X - \bar{X}$
2	−4
4	−2
6	0
8	2
10	4
	0

$\bar{X} = \frac{\Sigma X}{N} = \frac{30}{5} = 6$

$$\Sigma(X - \bar{X}) = 0$$

6 ① # ② 1 2

Mode ← |-7| = 7
→ |7| = 7.

absolute → positive → Mode [11]

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⑥ Sum of absolute deviation of all observation is minimum when taken about Median.

X	$\lvert X-\bar{X}\rvert$	$\lvert X-\text{Median}\rvert$
2	+5.2	+5
3	+4.2	+4
7	+0.2	0
9	1.8	2
15	7.8	8
	19.2	19

$\bar{X} = \frac{36}{5} = 7.2$

Median = 7

$$\boxed{\Sigma\,|x-\text{Median}| = \text{Minimum}}$$

$\Sigma\,|X-\bar{X}| = 19.2$

$\Sigma\,|X-\text{Median}| = 19.$

Properties of AM, GM and HM:

① If all observation are same then AM, GM and HM are also same.

Eg. 8, 8, 8, 8, 8

$$\boxed{AM = GM = HM} = 8$$

② Relationship between AM, GM and HM.

Case 1: For 2 numbers.

$$AM = \frac{a+b}{2}$$

$$GM = \sqrt{ab}$$

$$HM = \frac{2ab}{a+b}$$

$$\boxed{GM^2 = AM \times HM}$$

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Case 2:→ for any number 2, 3, 4, 5 . . . .

$$\boxed{AM \geq GM \geq HM}$$

(3) Application of GM and HM

- GM and HM are used for finding average rate and Average interest.

Avg rate / Avg interest → GM / HM

(4) Problems of Average Speed

Case I: distance is same = Use HM

Case II: time is same = Use AM

Case III: dist & time both are diff^n = Avg speed = $\frac{\text{Total dist}}{\text{Total time}}$

$$\text{Speed} = \frac{\text{Dist}}{\text{Time}}$$

Avg rate → GM / HM

(5) Combined Harmonic Mean

(1) (2)

$N_1 =$ $N_2 =$

$H_1 =$ $H_2 =$

$$H_{12} = \frac{N_1 + N_2}{\frac{N_1}{H_1} + \frac{N_2}{H_2}}$$

Average speed

dist same → HM

time same → AM

dist & time both are diff^n → Speed = $\frac{\text{Total dist}}{\text{Total time}}$

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Continuous Series

Inclusive Series
It includes upper limit & lower limit in the class interval.

eg: Inclusive → Exclusive

0 – 4	-0.5 – 4.5
5 – 9	4.5 – 9.5
10 – 14	9.5 – 14.5

Exclusive Series
It does not includes upper limit in that class. Upper limit is included in next class

eg. Exclusive

0 – 5
5 – 10
10 – 15

Mode:→ • Modes can not be defined uniquely.
Mode का सही उत्तर हमें नहीं मिलता.
Mode can come one, twice or thrice.

- Nomode.
- Unimodal (one) data
- Bimodal (two) data
- Trimodal (Three) data

Continuous Series

Less than / Below
Upper limit less than CF

Below 5
↳ 0-15

CF F

More than / above.
lower limit More than C.F

Above 15
↳ 15-20

CF F then CF

Partition Values

→ The values dividing a distribution into equal Parts.
founded by Ogive Graph.

Median

Median (Me)

- Discrete data without freq.
  (i) First arrange in A·O
  $Me = \left[\frac{N+1}{2}\right]^{th}$ value

- Discrete data with freq
  Step1→ Find less than Cf
  Step2→ $\frac{N+1}{2}$ find
  Step3→ Check that $Cf \geq \frac{N+1}{2}$
  Step4→ Cf → x → Median

- Continuous Data
  Step1→ Find less than cf.
  Step2→ Find $\frac{n}{2}$
  Step3→ $Cf \geq n/2$
  Step4→ Cf → Class → Median Class

$Median = l_1 + \left[\frac{n/2 - Cf}{F}\right] \times H$

Quartile

Q_1 Q_2 Q_3

- Discrete data without freq
  (i) First arrange in A·O
  $Q_p = \left[\frac{(N+1)p}{4}\right]^{th}$ value

- Discrete data with freq
  Step1:→ Find less than cf
  Step2:→ Find $\left[\frac{(N+1)p}{4}\right]$
  Step3:→ Check that $Cf \geq \frac{(N+1)p}{4}$
  Step4:→ Cf → x → Quartile

- Continuous Data
  Step1→ Find less than cf.
  Step2→ Find $\frac{np}{4}$
  Step3→ $Cf \geq np/4$
  Step4→ Cf → Class → Quartile Class

$Quartile_p = l_1 + \left[\frac{np/4 - Cf}{F}\right] \times H$

Decile

D_1 D_2 D_3 D_9

- Discrete data without freq.
  (i) First arrange in A·O
  $D_p = \left[\frac{(N+1)p}{10}\right]^{th}$ value

- Discrete data with freq
  Step1:→ Find less than cf
  Step2:→ Find $\left[\frac{(N+1)p}{10}\right]$
  Step3:→ Check that $Cf \geq \left[\frac{(N+1)p}{10}\right]$
  Step4:→ Cf → x → decile

- Continuous Data
  Step1:→ Find less than c.f.
  Step2→ Find $\frac{np}{10}$
  Step3→ $Cf \geq np/10$
  Step4→ Cf → Class → Decile Class

$Decile_p = l_1 + \left[\frac{np/10 - Cf}{F}\right] \times H$

Percentile

P_1 P_2 P_3 P_{99}

- Discrete data without freq.
  (i) First arrange in A·O
  $P_p = \left[\frac{(N+1)p}{100}\right]^{th}$ value

- Discrete data with freq
  Step1:→ Find less than c.f.
  Step2:→ Find $\left[\frac{(n+1)p}{100}\right]$
  Step3:→ Check that $Cf \geq \left[\frac{(n+1)p}{100}\right]$
  Step4:→ Cf → x → Percentile

- Continuous Data
  Step1→ Find less than c.f.
  Step2:→ Find $\frac{np}{100}$
  Step3:→ $Cf \geq np/100$
  Step4:→ Cf → Class → Percentile Class

$Percentile_p = l_1 + \left[\frac{np/100 - Cf}{F}\right] \times H$

It is an Property,

* Median = $Q_2 = D_5 = P_{50}$
always same

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AM > GM > HM → diff^n No.

AM = GM = HM → Same no.

AM ≥ GM ≥ HM → nothing mentioned [relation]

कोई भी -ve no हो → we cannot find GM
[Bcz -ve का root नहीं निकाल सकते]

Linear Relationship
($y = a + bx$)
↳ Change of scale & Change of Origin
→ Affects Mean, Median & Mode.

Combined Geometric Mean.

$GM_{xy} = GM_x \times GM_y$.

$GM_{x/y} = \frac{GM_x}{GM_y}$

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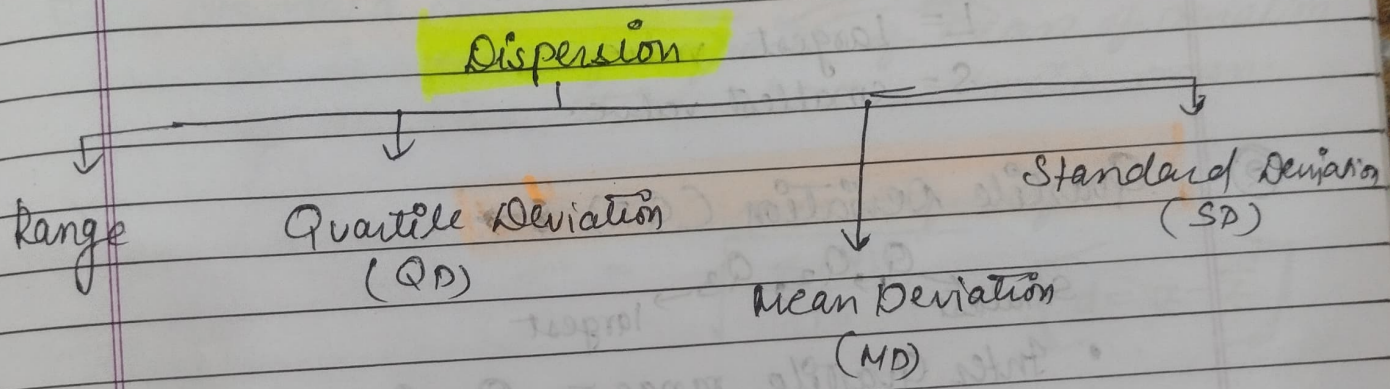
⇒ Dispersion :→

- It measures variation among data.



disper → बिखरना
dispersion → आपके data में कितना variation है।

More Concitent → variation कम (CV)
less Concitent → variation ज्यादा (CV)

• range निकाला जाते time frequency ही ~~आती~~ देखनी.

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① Range :→

• $R = L - S$

• Coefficent of range $= \frac{L-S}{L+S} \times 100$

L = Largest value
S = smallest value.

② Quartile Deviation (QD) :→

Q_1, Q_2, Q_3
Smallest ← → largest

• Inter Quartile range $= Q_3 - Q_1$

• $QD = \frac{Q_3 - Q_1}{2}$

• Coefficent of $QD = \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$

③ Mean Deviation (MD) :→

It is arithmetic mean of absolute deviation of all observation about central value.

• Discrete data without frequency.

$$M.D. = \frac{\Sigma |d|}{N}$$

d → deviation (diff^n)
$d = X - A$
A = Mean / Median / Mode

• Descrete data with frequency

$$MD. = \frac{\Sigma F|d|}{\Sigma F}$$

• Continuous data

$$M.D = \frac{\Sigma F|d|}{\Sigma F}$$

* About movement & dispersion

deviation → difference
absolute → +ve → (| |) → mode.

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Coefficient of MD = $\frac{MD}{A} \times 100$

(4) **Standard Deviation (SD) :→** $\boxed{SD = \sigma}$

It is root of squares of mean of deviation of all observation about arithmetic mean.

- Discrete data without frequency

$$S.D = \sigma = \sqrt{\frac{\Sigma (d)^2}{N}} \qquad \boxed{d = x - \bar{x}}$$

- Discrete data with frequency

$$S.D = \sigma = \sqrt{\frac{\Sigma f d^2}{\Sigma f}}$$

- Continuous data

$$S.D = \sigma = \sqrt{\frac{\Sigma f d^2}{\Sigma f}}$$

A.M ($\bar{X}$)	M.D	S.D (σ)
$\bar{X} = \frac{\Sigma x}{N}$	$MD = \frac{\Sigma \lvert d \rvert}{N}$	$\sigma = \sqrt{\frac{\Sigma (d)^2}{N}}$
$\bar{X} = \frac{\Sigma fx}{\Sigma f}$	$MD = \frac{\Sigma f \lvert d \rvert}{\Sigma f}$	$\sigma = \sqrt{\frac{\Sigma f d^2}{\Sigma F}}$
$\bar{X} = \frac{\Sigma fx}{\Sigma f}$	$MD = \frac{\Sigma f \lvert d \rvert}{\Sigma f}$	$\sigma = \sqrt{\frac{\Sigma f d^2}{\Sigma f}}$

Variance $= \sigma^2$

C.V = Coefficient of Variance

$= \frac{\sigma}{\bar{x}} \times 100$

Properties of Dispersion :-

① If all observations are same then range, QD, MD, SD are also same.

5, 5, 5, 5, 5 $\quad \bar{x} = \text{Median} = \text{Mode} = GM = HM = 5$

$R = MD = SD = QD = 0$

② For two numbers ⓐ and ⓑ

$\text{Range} = b - a$

$\sigma\ (S.D) = \frac{b-a}{2} = \frac{\text{Range}}{2}$

③ Measures of dispersion

- Absolute measure → Have some unit → Range, MD, SD, QD
- Relative Measure → no unit → Coeff^n of Range, MD, SD, & CV

④ Application of CV

Less C.V. ⟶ More Consistent, More stable

$$\bar{X}_{12} = \frac{\bar{X}_1 N_1 + \bar{X}_2 N_2}{N_1 + N_2}$$

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(5) Combined Standard Deviation

①	②	③
N_1	N_2	$N_1 + N_2$
$\bar{X}_1$	$\bar{X}_2$	$\bar{X}_{12}$
σ_1	σ_2	σ_{12}

$$\sigma_{12} = \sqrt{\frac{N_1(\sigma_1^2 + d_1^2) + N_2(\sigma_2^2 + d_2^2)}{N_1 + N_2}}$$

$d_1 = \bar{X}_{12} - \bar{X}_1$ & $d_2 = \bar{X}_{12} - \bar{X}_2$

(6) Change of scale and change of origin.

[× ÷] [+ −]

Central Tendency

↓

AM, Median, Mode

↓

(× ÷) Scale ✓

(+ −) Origin ✓

↓

$Y = a + bx$

$\bar{Y} = a + b\bar{x}$

$Me_y = a + b \cdot Me_y$

$Mo_y = a + b \cdot Mo_y$

Put the value

Dispersion

↓

Range, SD, MD, QD

↓

(× ÷) Scale ✓

Origin ✗

↓

$Y = a + bx$

$R_y = |b| \cdot R_x$

$\sigma_y = |b| \cdot \sigma_x$

$MD_y = |b| \cdot MD_x$

$QD_y = |b| \cdot QD_y$.

mode b

↓

|b|