CHAPTER 3

Equations

Equations

An equation is defined as a mathematical statement of equality.

Types of Equations

a) Linear equation in one variable.

b) Linear simultaneous equations in 2 or 3 variables.

c) Quadratic equations.

d) Cubic equations.

e) Bi-quadratic equations.

f) Exponential equations.

Quadratic Equations

- A quadratic equation is defined as polynomial equation of degree 2.
- A quadratic equation can be expressed in the following general form:

$$ax^2 + bx + c = 0;\ (a \neq 0)$$

- A quadratic equation can also be expressed in the factor form as follows:

$$a(x-\alpha)(x-\beta) = 0$$

  Here, α and β are the roots or solutions of quadratic equations.

- The general solution of the quadratic equation can be obtained as follows:

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$\text{Sum of roots} = \alpha + \beta = -\frac{b}{a}$$

- $$\text{Product of roots} = \alpha\beta = \frac{c}{a}$$

Structure of Quadratic Equations

If Sum (S) (α + β) and Product (P) (αβ) of the roots are known, then the quadratic equation is $x^2 - Sx + P = 0$

Sign of Roots of a Quadratic Equation

- When c=0, one root of the equation must be 0.
- When b and c are 0, then both the roots must be 0.
- If a, b, c all are of same sign, both roots are negative.
- If a and c are of same sign, opposite to that of b, then both the roots will be positive.
- If a and c are of opposite signs, one root is positive and another root is negative.

Nature of Roots

The expression "$b^2 - 4ac$" is called the "Discriminant (D)" of the quadratic equation.

- When D > 0, Roots are real and distinct.
- When D = 0, Roots are real and equal.
- When D < 0, Roots are imaginary.
- When D ≥ 0, Roots are real.
- When D is a perfect square, Roots are real, rational and unequal.
- When D is not a perfect square, Roots are real, irrational and unequal.
- If roots are equal use $b^2 = 4ac$.
- If roots are reciprocal of each other, use a = c
- If roots are equal but of opposite sign, use b = 0
- If roots are reciprocal but opposite in sign, use c = -a

Note

- Irrational roots will always appear in conjugate pairs.

  $\alpha = \left(a - \sqrt{b}\right)$ and $\beta = \left(a + \sqrt{b}\right)$

- Imaginary roots will always appear in conjugate pairs

  $\alpha = \left(a - ib\right)$ and $\beta = \left(a + ib\right)$

Cubic Equations

- A cubic equation is a polynomial equation of degree 3, and the general form is represented as follows:

  $ax^3 + bx^2 + cx + d = 0;\ (a \neq 0)$

- The factor form of a cubic equation is given as follows:

  $a(x-\alpha)(x-\beta)(x-\gamma) = 0$

  Here, α, β, and γ are the roots or solutions of the cubic equation.

- Sum of roots = α + β + γ = -b/a
- Product of the roots = αβγ = -d/a

Bi-Quadratic Equations

- A bi-quadratic equation is a polynomial of degree 4, and the general form is represented as follows:

  $ax^4 + bx^3 + cx^2 + dx + e = 0;\ (a \neq 0)$

- The factor form of a cubic equation is given as follows:

  $(x-\alpha)(x-\beta)(x-\gamma)(x-\delta) = 0$

  Here, α, β, γ and δ are the roots or solutions of the bi-quadratic equation.

- Sum of roots = α + β + γ + δ = -b/a
- Product of the roots = αβγδ = e/a

Equations – Solve for Roots

1. The roots of the equation $\frac{x+4}{4} + \frac{x-5}{3} = 11$

   a) 20

   b) 10

   c) 2

   d) None of these

2. 8 is the solution of the equation

   a) $\frac{x+4}{4} + \frac{x-5}{3} = 11$

   b) $\frac{x+4}{2} + \frac{x+10}{9} = 8$

   c) $\frac{x+24}{5} = 4 + \frac{x}{4}$

   d) $\frac{x-15}{10} + \frac{x+5}{5} = 4$

3. The equation $\frac{12x+1}{4} = \frac{15x-1}{5} + \frac{2x-5}{3x-1}$ is true for

   a) x = 1

   b) x =2

   c) x = 5

   d) x = 7

4. Pick up the correct value x for which $\frac{x}{0.5} - \frac{1}{0.05} + \frac{x}{0.005} - \frac{1}{0.0005} = 0$

   a) x = 0

   b) x =1

   c) x = 10

   d) None of these

5. $\frac{x}{4} = \frac{y}{3} = \frac{z}{2}$, 7x + 8y + 5z =62

   a) (4, 3, 2)

   b) (2, 3, 4)

   c) (3, 4, 2)

   d) (4, 2, 3)

6. $\frac{xy}{x+y} = 20$, $\frac{yz}{y+z} = 40$, $\frac{zx}{z+x} = 24$

   a) (120, 60, 30)

   b) (60, 30, 120)

   c) (30, 120, 60)

   d) (30, 60, 120)

7. $\frac{x}{p} + \frac{y}{q}$ = 2, x + y = p + q are satisfied by the values given by the pair.

   a) (x=p, y=q)

   b) (x =q, y=p)

   c) (x =1, y=1)

   d) None of these

8. $\frac{1}{3}(x + y) + 2z = 21,\ 3x - \frac{1}{2}(y + z) = 65,\ x + \frac{1}{2}(x + y - z) = 38$

a) (4, 9, 5)

b) (2, 9, 5)

c) (24, 9, 5)

d) (5, 24, 9)

9. $\frac{xy}{y-x} = 110, \frac{yz}{z-y} = 132, \frac{zx}{z+x} = \frac{60}{11}$

a) (12, 11, 10)

b) (10, 11, 12)

c) (11, 10, 12)

d) (12, 10, 11)

10. The satisfying values of x for the equation $\frac{1}{x+p+q} = \frac{1}{x} + \frac{1}{p} + \frac{1}{q}$ are

a) (p, q)

b) (-p, -q)

c) (p, -p)

d) (-p, q)

11. Given $\frac{\sqrt{x+4}+\sqrt{x-10}}{\sqrt{x+4}-\sqrt{x-10}} = \frac{5}{2}$. The value of x is:

a) 1

b) 331 / 5

c) 263 / 20

d) None of the above

12. The roots of the equation $\frac{x+\sqrt{12p-x}}{x-\sqrt{12p-x}} = \frac{\sqrt{p}+1}{\sqrt{p}-1}$ are:

a) $3p$

b) $-4p$

c) Either a) or b)

d) None of the above

13. If $\frac{x}{3x-y-z}=\frac{y}{3y-z-x}=\frac{z}{3z-x-y}=K$, then find the value of K .

a) 1

b) 3

c) 4

d) None of the above

14. Solve for (x,y): $47x-41y=53$ and $41x-47y=35$

a) (1,2)

b) (5,4)

c) (8,7)

d) None of the above

15. Solve for x: $\frac{1}{x-4}+\frac{1}{x-10}=\frac{1}{x-6}+\frac{1}{x-8}$

a) 6

b) 7

c) 8

d) None of the above

Linear Equations

16. The sum of two numbers is 52 and their difference is 2. The numbers are

a) 17 and 15

b) 12 and 10

c) 27 and 25

d) None of these

17. The diagonal of a rectangle is 5 cm and one of the side is 4 cm. Its area is

a) 20 sq. cm

b) 12 sq.cm

c) 10 sd.cm

d) None of these

18. The sum of the digits of a two digit number is 10. If 18 be subtracted from it the digits in the resulting number will be equal. The number is

    a) 37

    b) 73

    c) 75

    d) None of these numbers

19. The fourth part of a number exceeds the sixth part by 4. The number is

    a) 84

    b) 44

    c) 48

    d) None of these

20. Ten years ago the age of a father was four times of his son. Ten years hence the age of the father will be twice that of his son. The present age of the father and the son are.

    a) (50,20)

    b) (60,20)

    c) (55,25)

    d) None of these

21. The denominator of a fraction exceeds the numerator by 2. If 5 be added to the numerator the fraction increases by unity. The fraction is.

    a) $\frac{5}{7}$

    b) $\frac{1}{3}$

    c) $\frac{7}{9}$

    d) $\frac{3}{5}$

22. If a number of which the half is greater than $\frac{1}{5}$th of the number by 15 then the number is

    a) 50

    b) 40

    c) 80

    d) None of these

23. Monthly incomes of two persons are in the ratio 4 : 5 and their monthly expenses are in the ratio 7 : 9. If each saves Rs. 50 per month, find the monthly incomes
    a) (500, 400)
    b) (400, 500)
    c) (300, 600)
    d) (350, 550)

24. Y is older than x by 7 years 15 years back X's age was ¾ of Y's age. Their present ages are:
    a) (X = 36, Y = 43)
    b) (X = 50, Y = 43)
    c) (X = 43, Y = 50)
    d) (X = 40, Y = 47)

25. The demand and supply equations for a certain commodity are 4q + 7p = 17 and p = $\frac{q}{3}+\frac{7}{4}$, respectively where p is the market price and q is the quantity then the equilibrium price and quantity are :
    a) 2, $\frac{3}{4}$
    b) 3, $\frac{1}{2}$
    c) 5, $\frac{3}{5}$
    d) None of these

26. The difference of two positive integers is 3 and the sum of their squares is 89. Taking the smaller integer as x form a quadratic equation and solve it to find the integers. The integers are.
    a) (7, 4)
    b) (5, 8)
    c) (3, 6)
    d) (2, 5)

27. Five times of a positive whole number is 3 less than twice the square of the numbers. The number is
    a) 3
    b) 4
    c) -3
    d) 2

28. The area of a rectangular field is 2000 sq. m and its perimeter is 180 m. Form a quadratic equation by taking the length of the field as x and solve it to find the length and breadth of the field. The length and breadth are

    a) (205m, 80m)

    b) (50m, 40m)

    c) (40m, 50m)

    d) (a) and (b) both

29. Two squares have sides p cm and (p + 5) cms. The sum of their squares is 625 sq. cm. The sides of the squares are

    a) (10 cm, 30 cm)

    b) (12 cm, 25 cm)

    c) (15 cm, 20 cm)

    d) None of these

30. Divide 50 into two parts such that the sum of their reciprocals is 1/12. The numbers are

    a) (24, 26)

    b) (28, 22)

    c) (27, 23)

    d) (20, 30)

31. There are two consecutive numbers such that the difference of their reciprocals is 1/240. The numbers are

    a) (15, 16)

    b) (17, 18)

    c) (13, 14)

    d) (12, 13)

32. The sides of an equilateral triangle are shortened by 12 units 13 units and 14 units respectively and a right angle triangle is formed. The side of the equilateral triangle is

    a) 17 units

    b) 16 units

    c) 15 units

    d) 18 units

33. The sum of two irrational numbers multiplied by the larger one is 70 and their difference is multiplied by the smaller one is 12; the two numbers are

a) $3\sqrt{2}, 2\sqrt{3}$

b) $5\sqrt{2}, 3\sqrt{5}$

c) $2\sqrt{2}, 5\sqrt{2}$

d) None of these

34. The sum of a number and its positive square root is 6/25. Find the number.

a) 1/5

b) 1/25

c) 1/15

d) 1/35

35. Jayesh is as much younger to Anil as he is older to Prashant. If the sum of the ages of Anil and Prashant is 48 years, what is the age of Jayesh?

a) 20 years

b) 24 years

c) 30 years

d) Can't be determined

Quadratic Equations

36. If the roots of the equations $2x^2 + 8x - m^3 = 0$ are equal then value of m is

a) -3

b) -1

c) 1

d) -2

37. If α, ß be the roots of the equation $2x^2 - 4x - 3 = 0$ the value of $\alpha^2 + \beta^2$ is

a) 5

b) 7

c) 3

d) -4

38. If the sum of the roots of the quadratic equations $ax^2 + bx = c = 0$ is equal to the sum of the squares of their reciprocals then $\frac{b^2}{ac} + \frac{bc}{a^2}$ is equal to

    a) 2

    b) -2

    c) 1

    d) - 1

39. The equations $x^2 – (p +4)x + 2p + 5 = 0$ has equal roots the values of p will be

    a) ± 1

    b) 2

    c) ± 2

    d) – 2

40. The roots of the equation $x^2 + (2p – 1) x + p^2 = 0$ are real if

    a) $p \geq 1$

    b) $p \leq 4$

    c) $p \geq ¼$

    d) $p \leq ¼$

41. If x = m is one of the solutions of the equations $2x^2 + 5x – m = 0$ the possible values of m are

    a) (0, 2)

    b) (0, -2)

    c) (0, 1)

    d) (1, -1)

42. If L + M + N = 0 and L, M, N are rationals the roots of the equation $(M+N-L) x^2 + (N+L-M)x + (L+M-N) = 0$ are

    a) real and irrational

    b) real and rational

    c) imaginary and equal

    d) real and equal

43. If $p \neq q$ and $p^2 = 5p – 3$ and $q^2 = 5q – 3$ the equation having roots as $\frac{p}{q}$ and $\frac{q}{p}$ is

    a) $x^2 – 19x + 3 = 0$

    b) $3x^2 – 19x - 3 = 0$

    c) $3x^2 – 19x + 3 = 0$

    d) $3x^2 + 19x + 3 = 0$

44. If one rot of $5x^2 + 13x + p = 0$ be reciprocal of the other then the value of p is

a) -5

b) 5

c) 1/5

d) – 1/5

45. If the root of the equation $x^2 – 8x + m = 0$ exceeds the other by 4 then the value of m is

a) m = 10

b) m =11

c) m = 9

d) m = 12

46. The equation $\left(\frac{l-m}{2}\right)x^2 - \left(\frac{l+m}{2}\right)x + m = 0$ has got two values of x to satisfy the equation given as

a) $\left(1, \frac{2m}{l-m}\right)$

b) $\left(1, \frac{m}{l-m}\right)$

c) $\left(1, \frac{2l}{l-m}\right)$

d) $\left(1, \frac{1}{l-m}\right)$

47. Solving equation $(b-c)x^2 + (c – a)x + (a – b) = 0$ following roots are obtained

a) $\frac{a-b}{b-c}$, 1

b) (a-b)(a-c), 1

c) $\frac{b-c}{a-b}$, 1

d) none

48. The condition that one of $ax^2 + bx + c = 0$ the roots of is thrice the other is

a) $3b^2 = 16ca$

b) $b^2 = 9ca$

c) $3b^2 = - 16ca$

d) $b^2 = - 9ca$

49. The condition that the roots of the equations $ax^2 + bx + c = 0$ are in the ratio 4 : 5 is

a) $9b^2 = 20ac$

b) $20b^2 = 81ac$

c) $4b^2 = 54ac$

d) none

Discuss the nature of the roots of the following quadratic equations:

Working Codes: R = Real; E = Equal; D = Distinct & I = Imaginary

50. $3x^2 – 2x + 5 = 0$

a) I

b) R & D

c) R & E

d) E & I

51. $3x^2 + 7x + 2 = 0$

a) E & I

b) R & E

c) R & D

d) I

Find the values of m for which the following equations have equal roots:

52. $(1 + m) x^2 – 2 (1 + 3m) x + (1 + 8m) = 0$

a) 0, 3

b) 0

c) – 3

d) – 3, 0

53. The solution of the cubic equation

$x^3 – 6x^2 + 11x – 6 = 0$ is given by the triplet :

a) (-1, 1 -2)

b) (1, 2, 3)

c) (-2, 2, 3)

d) (0, 4, -5)

54. The cubic equation $x^3 – 2x^2 - x + 2 = 0$ has 3 roots namely:

a) (1, -1, 2)

b) (-1, 1, -2)

c) (-1, 2, -2)

d) (1, 2, 2)

55. x, (x – 4), (x + 5) are the factors of the left hand side of the equation.

a) $x^3 + 2x^2 - x – 2 = 0$

b) $x^3 + x^2 - 20x = 0$

c) $x^3 – 3x^2 - 4x + 12 = 0$

d) $x^3 – 6x^2 + 11x – 6 = 0$

56. The equation $3x^3 + 5x^2 = 3x + 5$ has got 3 roots and hence the factors of the left hand side of the equation

$3x^3 + 5x^2 - 3x - 5 = 0$ are

a) x – 1, x – 2, x – 5/3

b) x – 1, x + 1, 3x + 5

c) x + 1, x – 1, 3x – 5

d) x – 1, x + 1, x – 2

57. Factorize the left hand side of the equation $x^3 + 7x^2 – 21x – 27 = 0$ and the roots are as

a) (-3, -9 -1)

b) (3, -9, -1)

c) (3, 9, 1)

d) (-3, 9, 1)

58. The roots of $x^3 + x^2 – x – 1$ are

a) (-1, -1, 1)

b) (1, 1, -1)

c) (-1, -1, -1)

d) (1, 1, 1)

59. If $4x^3 + 8x^2 – x – 2 = 0$ then value of (2x + 3) is given by

a) 4, -1, 2

b) -4, 2, 1

c) 2, - 4, -1

d) None of these

60. The rational root of the equation $2x^3 - x^2 - 4x + 2 = 0$ is

a) $\frac{1}{2}$

b) $-\frac{1}{2}$

c) 2

d) – 2

61. Find the value of the given term: $5+\cfrac{1}{5+\cfrac{1}{5+\cfrac{1}{5+\cfrac{1}{5+.....}}}}$

a) $\frac{5+\sqrt{29}}{2}$

b) $\frac{5\pm\sqrt{29}}{2}$

c) $\frac{5-\sqrt{29}}{2}$

d) None of the above

62. If α, β are the roots of the equation $ax^2+bx+c=0$, then which of the following is/are true?

[1] If $\alpha = m+\sqrt{n}$, then $\alpha+\beta = 2m$

[2] If $\alpha = m+i\sqrt{n}$, then $\alpha-\beta = 2i\sqrt{n}$

[3] If $\alpha+\beta = 0$, then $c = a\alpha^2$

[4] If $\alpha\beta = 1$, then $\alpha^2\beta+\beta^2\alpha = -\frac{b}{c}$

a) 1 and 2

b) 1, 2 and 4

c) 1, 2 and 3

d) All of the above

63. If $\frac{x}{b}+\frac{b}{x}=\frac{a}{b}+\frac{b}{a}$ the roots of the equation are

a) a, b^2 / a

b) a^2, b / a^2

c) a^2, b^2 / a

d) a, b^2

64. Solving equation $6\left[\sqrt{\frac{x}{1-x}}+\sqrt{\frac{1-x}{x}}\right]$ = 13, following roots are obtained.

a) $\frac{4}{13}, \frac{9}{13}$

b) $\frac{-4}{13}, \frac{-9}{13}$

c) $\frac{4}{13}, \frac{5}{13}$

d) $\frac{6}{13}, \frac{7}{13}$

65. Solving equation $6x^4 + 11x^3 – 9x^2 -11x + 6 = 0$ following roots are obtained

a) $\frac{1}{2}$, -2, $\frac{-1\pm\sqrt{37}}{6}$

b) $-\frac{1}{2}$, 2, $\frac{-1\pm\sqrt{37}}{6}$

c) $\frac{1}{2}$, -2, $\frac{5}{6}, \frac{-7}{6}$

d) None

66. Find the roots of the equation: $2x^4 – 9x^3 + 14x^2 – 9x + 2 = 0$

a) 1, 2, ½

b) – 1, 2, ½

c) – 2, 1, ½

d) – ½, 1, 2

Consistency and inconsistency of linear equations:

67. The system of equation $5x – 4y = 7$ and $3x – 2y = 15$ have

(a) unique solution (b) infinite solution

(c) no solution (d) none

68. The system of equation $9x – 17y = 34$ and $36x – 68y = 115$ have

(a) unique Solution (b) infinite Solution

(c) no solution (d) none

69. The system of equation $2x + 3y = 7$ and $6x + 5y = 11$ have

    (a) unique solution (b) infinite solution

    (c) no solution (d) none

70. The system of equation $6x + 5y = 11$ and $9x + (15/2)y = 21$ have

    (a) unique Solution (b) infinite Solution

    (c) no solution (d) none

71. The system of equation 4x + 7y = 10 and 10x + (35/2)y = 25 have

    (a) unique solution (b) infinite solution

    (c) no solution (d) none

72. The value of k for which the system of equations: kx + 2y =5; 3x + y =1 have unique solution is:

    (a) $k \neq 6$ (b) k =4 (c) k =3 (d) k=1

73. The value of k for which the system of equations: kx + 2y =5; 3x + y =1 have no solution is:

    (a) k = 5 (b) k = 6 (c) k =3 (d) k=7

74. The value of k for which the system of equations: 7x - y =5; 21x - 3y =k have infinite solution is:

    (a) k = 4 (b) k = 15 (c) k =3 (d) k=7

75. Determine the values of a and b for which the following system of linear equations has consistent infinite many solutions: $2x – (a-4)y = 2b + 1$ and $4x – (a-1)y = 5b - 1$

    (a) a = -7, b =3 (b) a = 7, b =3 (c) a = -7, b = -3 (d) none