Ch-1

Ratio and Proportion, Indices, Logarithms

Ratio

→ Ratio is a comparison of the size of two or more quantities of the same kind by division.

→ If a and b are two quantities of the same kind (same unit), then the fraction $\frac{a}{b}$ is called the ratio of a to b. It is written as a : b

→ a : b → Consequent
↓
antecedent

Feature of Ratio

→ Both terms of a ratio can be multiplied or divided by the same (non-zero) number. Usually a ratio is expressed in lowest terms (or simplest form).

→ The order of the terms in a ratio is important

→ Ratio exists only between quantities of the same kind.

→ Quantities to be compared (by division) must be in the same units.

→ To compare two ratio, convert them into equivalent like fractions.

→ If a quantity increase or decrease in the ratio a : b

$$\boxed{\text{New quantity} = \frac{b}{a} \times \text{original quantity}}$$

The fraction by which the original quantity is multiplied to get a new quantity is called the factor multiplying ratio. (i.e $\frac{b}{a}$)

Q) Rounaq weight 56.7 kg. If he reduces his weight in the ratio 7:6. find new weight.

$\rightarrow$ $\frac{\cancel{56.7}^{8.1} \times 6}{\cancel{7}} = 48.6$ Ans.

Q) ~~Simplest~~ Simplify the ratio $\frac{1}{3} : \frac{1}{8} : \frac{1}{6}$

Sol. $\frac{1}{3} \times 24 : \frac{1}{8} \times 24 : \frac{1}{6} \times 24$

$8 : 3 : 4$

$2 \,|\, 3, 8, 6$
$3 \,|\, 3, 4, 3$
$1, 4, 1$

24

Properties of Ratio

> Inverse Ratio :- One ratio is the inverse of another if their product is 1. Thus b : a is the inverse of a : b and vice-versa.

> The ratio compounded of the two ratio a : b and c : d is ac : bd.

> Compounding Two or more ratios means multiplying them.

> A ratio compounded of itself is called it duplicate ratio.

$\boxed{a^2 : b^2}$ $\rightarrow$ is the duplicate ratio a : b.

$\boxed{a^3 : b^3}$ $\rightarrow$ is the triplicate ratio a : b.

$\boxed{\sqrt{a} : \sqrt{b}}$ $\rightarrow$ is the sub-duplicate ratio of a : b

$\boxed{\sqrt[3]{a} : \sqrt[3]{b}}$ $\rightarrow$ is the sub-~~duplicate~~ triplicate ratio of a : b

> If the ratio of two similar quantities can be expressed as a ratio of two integers, the quantities are said to be <u>commensurable</u>.

> Otherwise they are said to be incommensurable.

Example of <u>incommensurable quantities</u> $\boxed{\sqrt{3} : \sqrt{2}}$

> <u>Continue Ratio</u> :- is the relation or comparison between the magnitudes of three or more quantities of same kind.

> The continued ratio of three similar quantities a, b, c can be written as <u>a : b : c</u>.

Proportions

→ An equality of two ratio is called a proportion.

→ four quantities a, b, c are said to be in proportion if.

$$\boxed{\begin{matrix} a:b = c:d \\ a:b :: c:d \end{matrix}}$$

→ The quantities a, b, c, d are called terms of the proportion, a, b, c and d are called its first, second, third and fourth term respectively.

→ Terms of proportion can also be called as proportional.

→ Cross Product Rule: if $a:b = c:d$ are in proportion then $ad = bc$

Product of extremes = product of means

→ Continuous proportion :- Three quantities a, b, c of the same kind (in same units) are said to be in continuous proportion if $a:b = b:c$

$$\boxed{\frac{a}{b} = \frac{b}{c} \Rightarrow b^2 = ac}$$

a = 1st proportional

c = 3rd proportional

b = mean proportional.

→ In a ratio $a:b$, both quantities must be of same kind while in a proportion $a:b = c:d$. all the four quantities need not ~~of~~ be of the same type.

→ The first two quantities should be of the same kind and last two quantities should be of the same kind.

$\boxed{\text{Properties of Proportion}}$ Part IV

- **Invertendo**

$$\text{If } \frac{a}{b} = \frac{c}{d} \text{ then } \frac{b}{a} = \frac{d}{c} \left[\text{eg. } \frac{1}{2} = \frac{4}{8} \,:\, \frac{2}{1} = \frac{8}{4}\right]$$

- **Alternendo**

$$\text{If } \frac{a}{b} = \frac{c}{d} \text{, then } \frac{a}{c} = \frac{b}{d} \text{ or } \frac{d}{b} = \frac{c}{a} \left[\cancel{\text{eg. } \frac{1}{2} = \frac{4}{8}}\right]$$

- **Componendo**

$$\text{If } \frac{a}{b} = \frac{c}{d} \text{, then } \frac{a+b}{b} = \frac{c+d}{d}$$

- **Dividendo**

$$\text{If } \frac{a}{b} = \frac{c}{d} \text{, then } \frac{a-b}{b} = \frac{c-d}{d}$$

- **Componendo and Dividendo**

$$\text{If } \frac{a}{b} = \frac{c}{d} \text{, then } \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

- **Addendo**

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots$ then each of these ratio is equal to

$$\frac{a+c+e+\dots}{b+d+f+\dots}$$

This means that

$$\frac{a}{b} = \frac{a+c+e+\cdots c}{b+d+f+\cdots d};\ \frac{c}{d} = \frac{a+b+e+\cdots}{b+d+f+\cdots};\ \frac{e}{f} = \frac{a+c+e+\cdots}{b+d+f+\cdots}$$

eg.

$$\frac{1}{2} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$$

Addendo

$$\frac{1}{2} = \frac{1+3+4+5}{2+6+8+10} = \frac{13}{26} = \frac{1}{2}$$

$$\frac{3}{6} = \quad '' \quad = \quad '' \quad = \frac{1}{2}$$

$$\frac{4}{8} = \quad '' \quad = \quad '' \quad = \frac{1}{2}$$

$$\frac{5}{8} = \quad '' \quad = \quad '' \quad = \frac{1}{2}$$

- Subtrahendo

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \cdots$, then each of these rations is equal to

$$\frac{a-c-e-\cdots}{b-d-f-\cdots}$$

Eg.

$$\frac{1}{2} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$$

$$\frac{1}{2} = \frac{1-3-4-5}{2-6-8-10} = \frac{-\cancel{11}^{1}}{-\cancel{22}_{2}} = \frac{1}{2}$$

$$\frac{3}{6} = \quad '' \quad = \frac{-\cancel{11}^{1}}{-\cancel{22}_{2}} = \frac{1}{2}$$

$\frac{4}{8} = \frac{1-3-4-5}{2-6-8-10} = \frac{-\cancel{11}}{-\cancel{22}} = \frac{1}{2}$

$\frac{5}{10} =$ " $= \frac{-\cancel{11}}{-\cancel{22}} = \frac{1}{2}$

$\boxed{\text{Indices}}$

→ The word "Indices" is the plural of "index".

→ When a number is expressed in the form of a^n, a is called the base, and n is called the index / exponent / power.

$\boxed{\text{Integral components of a Real power}}$

- Positive integral power — for any real number and a positive integer n, a^n is defined as $a^n = a \times a \times a \times a \times - - - \times a$ (n times).
- Negative integral ~~power~~ power — for any real number a and a negative integer n, a^{-n} is defined as $a^{-n} = \frac{1}{a^n}$
- Zero power — for any real number a, a^0 is defined as $a^0 = 1$.

$\boxed{\text{Laws of indices}}$

- 1^{st} law

$$a^m \times a^n = a^{m+n}$$

Eg:- $2^4 \times 2^3 = 2^{4+3} = 2^7$

- 2^{nd} law

$$\frac{a^m}{a^n} = a^{m-n}$$

Eg:- $\frac{2^7}{2^2} = 2^{7-3}$

• 3rd law

$$(a^m)^n = a^{mn} = (a^n)^m$$

eg: $(2^4)^3 = 2^{4\times3} = 2^{12}$

• 4th law

$$(ab)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

eg: $(2\times4)^3 = 2^3\times4^3$

• 5th law

$a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}}$ i.e., $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

eg:

$\boxed{\text{Lo}\mathit{b}\text{arithm}}$ $\Rightarrow 2^3 = 8$

$2^8 = 3$

→ logarithms are used to simplify huge calculations $2^3 = 8$ is expressed in terms of Logarithms as ~~log~~ $\log_2 8 = 3$. It is read as log8 to the base 2 is 3.

Notes:-

- For any positive real number, a we know that $a^0 = 1$ and $a^1 = a$. Therefore, $\log_a 1 = 0$ and $\log_a a = 1$.
- If, in a questions, the base is not mentioned, it is considered to be 10.

Law of Logarithm

- 1st Law

$\log_a(mn) = \log_a m + \log_a n$

$\log_a m + \log_a n = \log_a(m \times n)$

- 2nd law

$\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

- 3rd law

$\log_a(m^n) = n \log_a m$

Eg. → $\log 4^2 = 2 \log 4$

• 4th law (Base change formula)

$\log_a m = \dfrac{\log_b m}{\log_b a}$ Eg. ⇒ $\log_4 10 = \dfrac{\log 10}{\log 4}$

• 5th law

$\dfrac{1}{\log_a m} = \log_m a$

Eg ⇒ $\log_2 4 = \dfrac{1}{\log_4 2}$

• 6th law

$a^{\log_a n} = n$

• 7th law

$\log_{a^q} n^p = \dfrac{p}{q} \log_a n$

Eg. = $\log_{4^2} 5^9 = \dfrac{9}{2} \log_4 5$