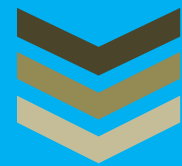


Referencer for Quick Revision



Foundation Course Paper-3: Business Mathematics and Logical Reasoning & Statistics

A compendium of subject-wise capsules published in the
monthly journal "The Chartered Accountant Student"



**Board of Studies
(Academic)
ICAI**

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CHAPTER 4 : TIME VALUE OF MONEY

At the foundation level with regards to Business Mathematics the topic Time Value of money is very important for students not only to acquire professional knowledge but also for examination point of view. Here in this capsule an attempt is made for solving and understanding the concepts of time value of money with the help of following questions with solutions.

Problems on Simple, Compound and Effective rate of interest

Simple interest = P.T.I., Where P = Principal, T = Time, I = Rate of Interest

1. A sum of money amount to ₹ 6,200 in 2 years and ₹ 7,400 in 3 years. The principal and rate of interest is

Solution: A sum of money in 2 years = $P + P.2.I = ₹ 6200$

A sum of money in 3 years = $P + P.3.I = ₹ 7400$

Interest in 1 year = ₹ 1200; Interest in 2 years = ₹ 2400

Amount = ₹ 6200, P = Principal = $6200 - 2400 = ₹ 3800$.

$$2400 = 3800 \times 2 \times \frac{I}{100}, I = \text{rate of interest} = 31.58\%$$

2. A sum of money doubles itself in 10 years. The number of years it would triple itself is

Solution: $2P = P + \frac{PTR}{100}$, $P = \frac{PTR}{PTR100}$; T = 10 then R = 10%

$$3P = P + \frac{PTR}{100} \text{ then } 2P = \frac{PTR}{100} \text{ and } R = 10\%$$

Time (T) = 20 years.

Amount = $P(1+i)^n$

Compound rate of interest = $P(1+i)^n - P = A - P$

where P = principle i = interest n = conversion period

3. The population of a town increases every year by 2% of the population at the beginning of that year. The number of years by which the total increase of population be 40% is

Solution: $1.4P = P(1+0.02)^n$

$$(1.02)^n = 1.4, n = 17 \text{ years (app)}$$

Depreciation (A) = $P(1-i)^n$

Where, A = Scrap Value, P = Original Cost, I = Depreciated at the rate, n = Number of years

4. A machine is depreciated at the rate of 20% on reducing balance. The original cost of the machine was ₹ 1,00,000 and its ultimate scrap value was ₹ 30,000. The effective life of the machine is

Solution: Here A = Scrap Value = 30,000 and

P = Original Cost = ₹ 1,00,000

$$30000 = 100000(1-0.2)^n$$

$$3/10 = 0.3 = (0.8)^n, n = 5.4 \text{ years}$$

5. The useful life of a machine is estimated to be 10 years and cost ₹ 10,000. Rate of depreciation is 10% p.a. The scrap value at the end of its life is

Solution: Here A = Scrap Value = ?,

P = Original cost = 10,000 n = 10, I = 10%

$$A = 100000(1-0.1)^{10} = 10000(0.9)^{10}$$

$$A = ₹ 3486.78$$

Effective rate of interest = $(1+i)^n - 1$

6. The effective rate of interest corresponding a nominal rate of 7% p.a convertible quarterly is

Solution: Effective rate of interest = $(1+i)^n - 1$, here n = 4,

$$i = 0.07/4 = 0.0175$$

$$= (1+0.0175)^4 - 1 = 1.07186 - 1 = 7.19\%$$

The difference between simple and compound interest for 2 years = Pi^2 , where P = Principal, i = interest

7. The difference between the S.I and the C.I on ₹ 2,400 for 2 years at 5% p.a is

Solution: The difference between simple and compound interest for 2 years = $2400(0.05)^2 = ₹ 6$.

The difference between simple and compound interest for 3 years = $3Pi^2 + Pi^3$, where P = Principal i = Interest

8. The difference between the S.I and the C.I on a certain money invested for 3 years at 6% p.a is ₹ 110.16 the principle is

Solution: The differences between simple and compound interest for 3 years

$$= 110.16 = P(3i^2 + i^3) = P(3 \times 0.06^2 + 0.06^3)$$

$$110.16 = P(0.0108 + 0.000216) = P(0.011016)$$

$$P = \frac{110.16}{0.011016} = ₹ 10,000$$

9. The annual birth and death rates per 1,000 are 39.4 and 19.4 respectively. The number of years in which the population will be doubled assuming there is no immigration or emigration is

Solution: Here given, birth rate per 1,000 = 39.4 and death rates per 1,000 = 19.4

difference = 20 % per 1000 population

$$\text{growth rate} = \frac{20}{1000} \times 100 = 2\%$$

Future population double

$$P = 1000, A = 2000, r = 2\%$$

$$2000 = 1000(1+0.02)^n$$

$$(1.02)^n = 2$$

Number of years = n = 35

10. What annual rate of interest compounded annually doubles an investment in 7 years?

(Given that $2^{1/7} = 1.104090$)

Solution: If the principal be P, $A_n = 2P$

$$\text{Since } A_n = P(1+i)^n$$

$$2P = P(1+i)^7$$

$$2^{1/7} = (1+i)$$

$$1.104090 = 1+i$$

$$I = 0.10409, \text{ Required rate of interest} =$$

$$10.41\% \text{ per annum}$$

11. Vidya deposited ₹ 60000 in a bank for two years with the interest rate of 5.5% p.a. How much interest she would earn? what will be the final value of investment?

Solution: Required interest amount is given by,

$$I = P \times it = ₹ 60,000 \times \frac{5.5}{100} \times 2 = ₹ 6,600$$

The amount value of investment is given by, $A = P + I = ₹ (60,000 + 6600) = ₹ 66,600$

12. Rajiv invested ₹ 75,000 in a bank at the rate of 8% p.a. simple interest rate. He received ₹ 135,000 after the end of term. Find out the period for which sum was invested by Rajiv.

Solution: We know $A = P + Pit = P(1+it)$

$$\text{i.e. } 135000 = 75000(1 + \frac{8}{100} \times t)$$

$$135000/75000 = \frac{100+8t}{100}$$

$$1.8 \times 100 - 100 = 8t$$

$$80 = 8t$$

$$t(\text{Time}) = 10 \text{ years}$$

13. Which is a better investment, 3.6% per year compounded monthly or 3.2% per year simple interest? Given that $(1+0.003)^{12} = 1.0366$.

Effective rate of interest $E = (1 + i)^n - i$

Solution: $i = 3.6/12 = 0.3\% = 0.003, n = 12$
 $E = (1 + i)^n - 1$
 $= (1 + 0.003)^{12} - 1 = 1.0366 - 1 = 0.0366$ or $= 3.66\%$

Effective rate of interest (E) 3.66% is more than the simple interest so the Effective rate of interest (E) is better investment than the simple interest 3.2% per year.

14. The C.I on ₹ 4,000 for 6 months at 12% p.a payable quarterly is

Solution: Here $P = ₹ 4000, n = 6/3 = 2, r = 0.12/4 = 0.03$
 Compound Interest $= [P(1+i)^n - P] = [4000(1+0.03)^2 - 4000] = ₹ 243.60$

Annuity applications

$F =$ Future value $= C.F. (1 + i)^n$ Where C.F = Cash flow
 $i =$ rate of interest, $n =$ time period

15. Ravi invest ₹ 5000 in a two-year investment that pays you 12% per annum. Calculate the future value of the investment

Solution: We know, $F =$ Future value $= C.F.(1 + i)^n$, Where C.F= Cash flow $= ₹ 5000, i =$ rate of interest $= 0.12, n =$ time period $= 2$

$F = ₹ 5000(1+0.12)^2 = ₹ 5000 \times 1.2544 = ₹ 6272.$

Annuity regular means: First payment at the end of the period.

Future value of the annuity regular $= A(n, i) = A \cdot \left[\frac{(1+i)^n - 1}{i} \right]$

Annuity regular: In annuity regular first payment/receipt takes place at the end of first period.

16. Find the future value of an annuity of ₹ 5000 is made annually for 7 years at interest rate of 14% compounded annually. [Given that $(1.14)^7 = 2.5023$]

Solution: Here annual payment $A = ₹ 5000, n = 7, i = 14\% = 0.14$

Future value of the annuity $= A(7, 0.14) = 5000 \cdot \left[\frac{(1+0.14)^7 - 1}{0.14} \right] = 5000 \left[\frac{(2.5023 - 1)}{0.14} \right] = ₹ 53653.57$

Future value of Annuity due or Annuity Immediate: When the first receipt or payment at the beginning of the annuity) it is called annuity due or annuity immediate.

17. ₹ 2000 is invested at the end of each month in an account paying interest 6% per year compounded monthly. What is the future value of this annuity after 10th payment? Given that $(1.005)^{10} = 1.0511$

Solution: Here $A = ₹ 2000, n = 10, i = 6\%$ per annum $= 6/12\%$ per month $= 0.005$

Future value of annuity after 10 months is given by

$A(n, i) = A \left[\frac{(1+i)^n - 1}{i} \right]$
 $A(10, 0.005) = 2000 \left[\frac{(1+0.005)^{10} - 1}{0.005} \right] = 2000 \left[\frac{(1.0511) - 1}{0.005} \right]$
 $= 2000 \times 10.22 = ₹ 20440$

Future value of the annuity regular or annuity

due $= A \left[\frac{(1+i)^n - 1}{i} \right] \times (1 + i)$

18. Swati invests ₹ 20,000 every year starting from today for next 10 years. Suppose interest rate 8% per annum compounded annually. Calculate future value of the annuity. Given that $(1 + 0.08)^{10} = 2.158925$

Solution: Calculate future value as though it were an ordinary annuity. Future value of the annuity as if it were an ordinary annuity

$= ₹ 20000 \left[\frac{(1+0.08)^{10} - 1}{0.08} \right]$
 $= ₹ 20000 \times 14.486563 = ₹ 289731.25$

Multiply the result by $(1 + i) = ₹ 289731.25 \times (1 + 0.08) = ₹ 312909.76$

19. What is the present value of ₹ 100 to be received after two years compounded annually at 10%.

Solution: Here $A_n = ₹ 100, i = 10\% = 0.1, n = 2$

Required present value $= \frac{A_n}{(1+i)^n} = \frac{100}{(1+0.1)^2} = \frac{100}{(1.21)} = ₹ 82.64$

Thus ₹ 82.64 shall grow to ₹ 100 after 2 years at 10% compounded annually.

20. Find the present value of ₹ 10000 to be required after 5 years if the interest rate be 9%. Given that $(1.09)^5 = 1.5386$.

Solution: Here $i = 0.09, n = 5, A_n = 10000$

Required present value $= \frac{A_n}{(1+i)^n} = \frac{10000}{(1+0.09)^5} = \frac{10000}{(1.5386)} = ₹ 6499.42$

Present Value of Annuity regular $= A = P(n, i) = A \cdot \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$,
 $P(n, i) = \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$

21. Soni borrows ₹ 5,00,000 to buy a car. If he pays equal instalments for 10 years and 10% interest on outstanding balance, what will be the equal annual instalment? Given $[P(10, 0.10) = 6.14457]$

Solution: We know, $A = \frac{V}{P(n, i)}$ Here $V = ₹ 500000, n = 10, I = 10\%$ p.a. $= 0.10$

Annual Instalment $= \frac{V}{P(n, i)} = ₹ \frac{5,00,000}{6.14457} = ₹ 8,1372.66$

22. If ₹ 10,000 is paid every year for ten years to pay off a loan. What is the loan amount if interest rate be 14% per annum compounded annually? Given $[P(10, 0.14) = 5.21611]$

Solution: $V = A \cdot P(n, i)$ Here $A = ₹ 10000, n = 10, i = 0.14$
 $V = 10000 \times P(10, 0.14)$
 $= 10000 \times 5.21611 = ₹ 52161.10$
 Therefore, the loan amount is ₹ 52161.10

23. Ram bought a Scooter costing ₹ 73000 by making a down payment of ₹ 3000 and agreeing to make equal annual payment for four years. How much would be each payment if the interest on unpaid amount be 14% compounded annually? Given $[P(4, 0.14) = 2.91371]$

Solution: In the present case we have present value of the annuity i.e. ₹ 70000 (73000-3000) and we have to calculate equal annual payment over the period of four years.

We know that, $V = A \cdot P(n, i)$ Here $n = 4$ and $I = 0.14$

$A = \frac{V}{P(n, i)} = \frac{70000}{P(4, 0.14)} = \frac{70000}{2.91371}$

Therefore, each payment $= ₹ 24024.35$

24. Suppose your Father decides to gift you ₹ 20,000 every year starting from today for the next six years. You deposit this amount in a bank as and when you receive and get 10% per annum interest rate compounded annually. What is the present value of this annuity?

Solution: For calculating value of the annuity immediate following steps will be followed. Present value of the annuity as if it were a

regular annuity for one year less i.e. for five years

$$= ₹ 20,000 \times P(5, 0.10)$$

$$= ₹ 20,000 \times 3.79079 = ₹ 75815.80$$

Add initial cash deposit to the value, ₹ (75815.80+20,000) = ₹ 95815.80

Sinking Fund: Interest is computed at end of every period with specified interest rate.

25. How much amount is required to be invested every year so as to accumulate ₹ 5,00,000 at the end of 10 years if interest is compounded annually at 10%? Given A. (10, 0.1) = 15.9374248

Solution: Here $A = 500000, n = 10, A(n, i) = \left[\frac{(1+i)^n - 1}{i} \right] =$
 $\left[\frac{(1+0.1)^{10} - 1}{0.1} \right] = 15.9374248$
 since $A = P \cdot A. (n, i)$
 $500000 = P \cdot A. (10, 0.1) = P \times 15.9374248$
 $P = \left[\frac{500000}{15.9374248} \right] = ₹ 31372.70$

26. ABC Ltd. wants to lease out an asset costing ₹ 360000 for a five year period. It has fixed a rental of Rs.105000 per annum payable annually starting from the end of first year. Suppose rate of interest is 14% per annum compounded annually on which money can be invested by the company. Is this agreement favorable to the company?

Solution: First, we have to compute the present value of the annuity of ₹ 105000 for five years at the interest rate of 14% p.a. compounded annually.

The present value V of the annuity is given by
 $V = A \cdot P(n, i) = 105000 \times P(5, 0.14)$
 $= 105000 \times 3.43308 = ₹ 360473.40$

which is greater than the initial cost of the asset and consequently leasing is favourable to the lessor.

27. A company is considering proposal of purchasing a machine either by making full payment of ₹ 4000 or by leasing it for four years at an annual rate of ₹ 1250. Which course of action is preferable if the company can borrow money at 14% compounded annually?

Solution: The present value V of annuity is given by

$$V = A \cdot P(n, i) = 1250 \times P(4, 0.14)$$

$$= 1250 \times 2.91371 = ₹ 3642.11$$

which is less than the purchase price, and consequently leasing is preferable.

28. A machine can be purchased for ₹ 50000. Machine will contribute ₹ 12000 per year for the next five years. Assume borrowing cost is 10% per annum compounded annually. Determine whether machine should be purchased or not.

Solution: The present value of annual contribution

$$V = A \cdot P(n, i)$$

$$= 12000 P(5, 0.10) = 12000 \times 3.79079$$

$$= ₹ 45489.48$$

which is less than the initial cost of the machine. Therefore, machine must not be purchased.

29. A machine with useful life of seven years costs ₹ 10000 while another machine with useful life of five years costs ₹ 8000. The first machine saves labour expenses of ₹ 1900 annually and the second one saves labour expenses of ₹ 2200 annually. Determine the preferred course of action. Assume cost of borrowing as 10% compounded per annum.

Solution: The present value of annual cost savings for the first machine
 $= ₹ 1900 \cdot P(7, 0.10)$
 $= ₹ 1900 \times 4.86842 = ₹ 9250$

cost of machine being Rs.10000 it costs more by ₹ 750 than it saves in terms of labour cost.

The present value of annual cost savings of the second machine = ₹ 2200.P(5,0.10) = ₹ 2200 x 3.79079 = ₹ 8339.74

Cost of the second machine being ₹ 8000, effective savings in labour cost is ₹ 339.74. Hence the second machine is preferable.

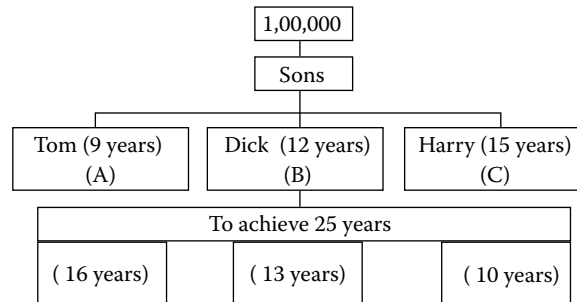
30. An investor intends purchasing a three year ₹ 1000 par value bond having nominal interest rate of 10%. At what price the bond may be purchased now if it matures at par and the investor requires a rate of return of 14%?

Solution: Present value of the bond = $\frac{100}{(1+0.14)^1} + \frac{100}{(1+0.14)^2} +$
 $\frac{100}{(1+0.14)^3} + \frac{1000}{(1+0.14)^3}$
 $= 100 \times 0.87719 + 100 \times 0.769467 + 100 \times 0.674972$
 $+ 1000 \times 0.674972$
 $= 87.719 + 76.947 + 67.497 + 674.972 = 907.125$

Thus the purchase value of the bond is ₹ 907.125

31. Johnson left ₹ 1,00,000 with the direction that it should be divided in such a way that his minor sons Tom, Dick and Harry aged 9, 12 and 15 years should each receive equally after attaining the age 25 years. The rate of interest being 3.5%, how much each son receives after getting 25 years old?

Solution: Given problem can be explained as



$$A \left(1 + \frac{3.5}{100} \right)^{16} = B \left(1 + \frac{3.5}{100} \right)^{13} = C \left(1 + \frac{3.5}{100} \right)^{10}$$

$$A(1.035)^{16} = B(1.035)^{13} = C(1.035)^{10}$$

$$A(1.035)^6 = B(1.035)^3 = C \text{-----(I)}$$

$$A : B : C = 1 : (1.035)^3 : (1.035)^6$$

$$A + B + C = 100000$$

$$x + x(1.035)^3 + x(1.035)^6 = 100000$$

$$x [1 + (1.035)^3 + (1.035)^6] = 10,0000$$

$$x(3.337973) = 1,00,000 \text{ then } x = ₹ 29958.30 \text{ (A's share)}$$

$$B's \text{ share} = 29958.30 (1.035)^3 = ₹ 33215.30$$

$$C's \text{ share} = 29958.30 (1.035)^6 = ₹ 36826.40$$

32. A machine costs ₹ 5,20,000 with an estimated life of 25 years. A sinking fund is created to replace it by a new model at 25% higher cost after 25 years with a scrap value realization of ₹ 25000. what amount should be set aside every year if the sinking fund investments accumulate at 3.5% compound interest p.a.?

Solution: Cost of new machine = $5,20,000 \times \frac{125}{100} = ₹ 6,50,000$, Scrap value = ₹ 25,000

For new machine = $650000 - 25000 = ₹ 6,25,000$.

Here = ₹ 6,25,000, n = 25, i = 3.5% = 0.035

$$6,25,000 = P \cdot \left[\frac{(1+i)^n - 1}{i} \right] = P \cdot \left[\frac{(1+0.035)^{25} - 1}{0.035} \right]$$

$$6,25,000 = P [38.95] \text{ then } P = \frac{625000}{38.95} = ₹ 16046.27$$

Foundation Paper 3: Logical Reasoning Questions with explanations

At the Foundation level, students are expected to inculcate/evolve logical thinking and reasoning skills to further develop their analytical skills. This section attempts to capture basic techniques in sequential thinking as the underlying concept to solve problems. Here are a few Logical Reasoning Questions with explanations to get you psyched!

Chapter : 9 Number Series, Coding and decoding and Odd man out series

These questions deal in which series or letters in some orders and follows certain pattern throughout.

I. Find missing term of the series

(1) 101, 102, 106, 115, 131, 176, ?

(a) 212 (b) 220 (c) 211 (d) 235

Explanation: Answer: (a)

The pattern of the series by adding $+1^2, +2^2, +3^2, +4^2, +5^2, +6^2$,

So missing term is $176 + 6^2 = 212$.

(2) 3, 10, 29, 66, 127, ?

(a) 164 (b) 187 (c) 216 (d) 218

Explanation: Answer: (d)

The pattern of the series is $1^3+2, 2^3+2, 3^3+2, 4^3+2, 5^3+2, 6^3+2$

So missing number is, $6^3+2 = 216 + 2 = 218$.

(3) 8, 13, 21, 32, 46, 63, 83, ?

(a) 104 (b) 106 (c) 108 (d) 110

Explanation: Answer: (b)

The pattern of the series is $+5, +8, +11, +14, +17, +20, +23$

So missing number is $83 + 23 = 106$

(4) 3, 4, 4, 6, 12, 15, 45, ?, 196

(a) 42 (b) 49 (c) 43 (d) 40

Explanation: Answer: (b)

The pattern of the series is $3+1, 4 \times 1, 4+2, 6 \times 2, 12+3, 15 \times 3, 45+4, 49 \times 4$,

So missing term is $= 45 + 4 = 49$

(5) 10, 12, 22, 34, 56, 90, ?

(a) 146 (b) 147 (c) 136 (d) 156

Explanation: Answer: (a)

Each term in the series, except the first two terms, is the sum of preceding two terms

The right answer $56+90 = 146$

(6) 4, 9, 19, 39, ?, 159, 319

(a) 40 (b) 41 (c) 78 (d) 79

Explanation: Answer: (d)

Each number of the series is one more than the twice the preceding number

Therefore, missing term $= 39 \times 2 + 1 = 79$

(7) 7, 15, 29, 59, 117, ?

(a) 238 (b) 235 (c) 120 (d) 155

Explanation: Answer: (b)

The pattern is $7 \times 2 + 1, 15 \times 2 - 1, 29 \times 2 + 1, 59 \times 2 - 1, 117 \times 2 + 1$

So missing term is $= 117 \times 2 + 1 = 235$

II. Find missing term of the letter series

(8) DBA, IDE, NFI, SHO, ?

(a) XJU (b) XYU (c) XUV (d) XUY

Explanation: Correct Option: Answer: (a)

The first letter of the series is

$D \rightarrow +5 \quad I \rightarrow +5 \quad N \rightarrow +5 \quad S \rightarrow +5 \quad X$

The second letter of the series $B \rightarrow +2 \quad D \rightarrow +2 \quad F \rightarrow +2 \quad H \rightarrow +2 \quad J$

The third letters of the series are pattern

A - E - I - O - U > Vowels

So missing letter series is XJU

(9) cccaa_bb_cc_aa_bb_

(a) abcab (b) babda (c) badna (d) bdanb

Explanation: Answer: (a)

The pattern of the series is ccc, aaa, bbb, ccc, aaa follows.

(10) m_nv_n_a_n_a_ma_

(a) aamvnn (b) aanvmm (c) vamaal (d) vanmak

Explanation: Answer: (a)

The series man and van repeated

(11) In a certain language 'BISLERI' is written as 'CHTKFQJ' and 'AQUA' is written as 'BPVZ'. How is 'COMPUTER' written in the same Code?

(a) DNNOVSFQ (b) DNNVOSFX (c) DNNOVVSX

(d) DNONVSEF

Explanation: Answer: (A)

B •+1 c	I •-1 H	S •+1 T	L •-1 K	E •+1 F	R •-1 Q	I •+1 J
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A •+1 B	Q •-1 P	U •+1 V	A •-1 Z
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C •+1 D	O •-1 N	M •+1 N	P •-1 o	U •+1 V	T •-1 S	E •+1 F	R •-1 Q
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So "COMPUTER" is coded as 'DNNOVSFQ'

(12) If 'BROTHER' is coded as 2456784. 'SISTER' is coded as 919684, what is the code for 'ROBBERS'?

(a) 4562684 (b) 9245784 (c) 4522849 (d) 4652684

Explanation: Answer: (c)

B •2	R •4	O •5	T •6	H •7	E •8	R •4
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S •9	I •1	S •9	T •6	E •8	R •4
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R •4	O •5	B •2	B •2	E •8	R •4	S •9
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Find odd one of the following series

(13) (a) 144 (b) 169 (c) 288 (d) 324

Explanation: Answer: (c), All others are perfect square numbers except 288

LOGICAL REASONING ||

(14) (a) 73 (b) 53 (c) 87 (d) 23

Explanation: Answer (c), Except 87 all others are prime numbers

(15) (a) 4867 (b) 5555 (c) 6243 (d) 6157

Explanation: Answer (d)

$4867 = 4+8+6+7 = 25$, which is divisible by 5

$5555 = 5+5+5+5 = 20$, which is divisible by 5

$6243 = 6+4+2+3 = 15$, which is divisible by 5

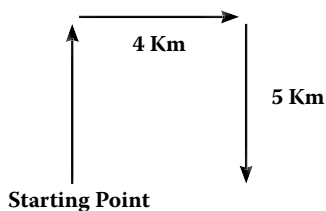
$6157 = 6+1+5+7 = 19$, which is not divisible by 5

Chapter 10. Direction tests: In this test, the questions consist of a sort of direction puzzle. A successive follow-up of direction is formulated and the student is required to ascertain the final direction. The test is meant to judge the candidate's ability to trace and follow correctly and sense the direction correctly.

(1) A man walks 6 km North, turns right and walks 4 km, again turns right and walks 5 km, in which direction is he facing now?

(a) South (b) North (c) East (d) West

Explanation:

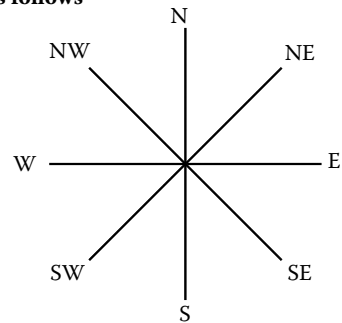


Answer: (a) South

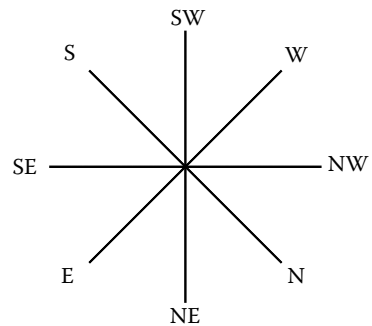
(2) If South-East becomes North, North-East becomes West and all the rest of the directions are changed in same manner, what will be direction of the East?

(a) North-West (b) South (c) South-East (d) South-West

Explanation: According to question the direction of the diagram as follows



After changing the directions

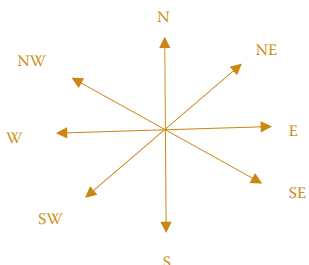


Now from the above diagrams North-West will be the direction for East.

Foundation Paper 3: Logical Reasoning Questions with explanations

At the Foundation level, students are expected to inculcate/evolve logical thinking and reasoning skills to further develop their analytical skills. This section attempts to capture basic techniques underlying concept of direction-related problems. Here are a few Logical Reasoning Questions with explanations to get you psyched!

CHAPTER 10: DIRECTION SENSE TESTS

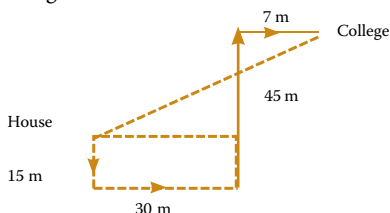


1. Gopal goes 15 m south from his house, turns left and walks 30 m, again turns left and walks 45 m, then turns right and walks 7 m to reach the college. In which direction is the college from his house?

(a) North-East (b) West (c) East (d) North

Explanation: Answer (a)

According to the information stated in the question, direction diagram can be drawn as follows.

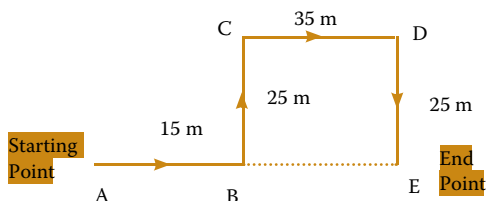


So it's clear from the diagram that college is North -East direction from Gopal's house

2. Ram start moving from a point, facing in East direction. After walking 15 m, he turned to his left and walked 25m, before turning to his right. Then, he walked a distance of 35 m, then turned to his right and stop after walking further a distance of 25 m. Find how far Ram is from his starting point.

(a) 20 m (b) 50 m (c) 15 m (d) 25 m

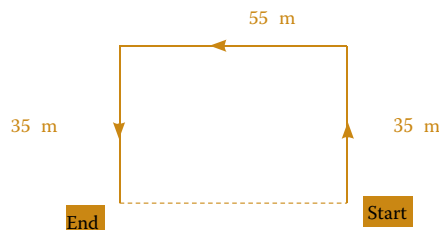
Explanation: Answer (b), the direction map of Ram's walk can be drawn as,



The distance between the starting point and end point is $AB + BE = 15 + 35 = 50$ m.

3. Facing towards North, Ravi walks 35 m. He then turns left and walks 55 m. He again turns left and walks 35 m. How far is from original position and towards which direction.
- (a) 30 m, North (b) 20 m, East (c) 55 m, West (d) 20 m, South

Explanation: Answer (c)



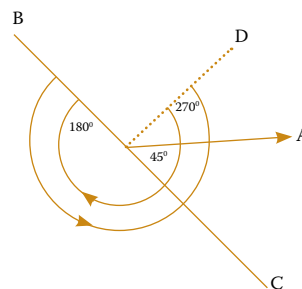
From the figure it is clear that, Ravi is 55 m away in West direction from his original position.

4. A man is facing towards East and turns through 45° clockwise again 180° clockwise and then turns through 270° anti-clockwise. In which direction is he facing now?

(a) West (b) North- East (c) South (d) South-West

Explanation: Answer (b)

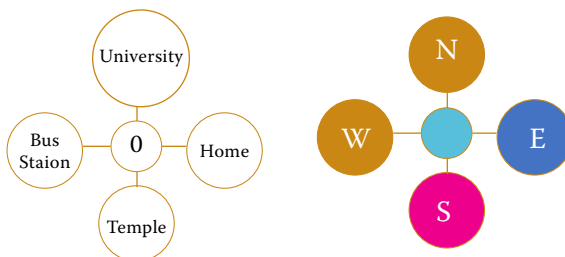
As shown in figure, the man initially faces in the direction of OA. On moving 45° clockwise, the man faces the direction OB. On further moving 180° clockwise, he faces in the direction of OC. Finally on moving 270° anti-clockwise, he faces the direction OD, which is North-East.



5. Kamal wants to go to university which is situated in a direction opposite to that of a temple. He starts from his house, which is in the East and comes at a four-ways place. His left side road goes to the temple and straight in front is the Bus Station. In which direction is university is located?

(a) North (b) North- East (c) South (d) East

Explanation: Answer (c)



Kamal comes from East towards West. He reached O (four-way place). Now university will not in front or left. It will be towards the right, so it will be north direction.

CHAPTER 11. SEATING ARRANGEMENTS

The process of making group of people to sit as per a prefixed manner is called seating arrangement these questions, some conditions are given on the basis of which students are required to arrange objects, either in a row or on in circular order.

1. Six Children A, B, C, D, E and F are sitting in a row facing towards North. C is sitting between A and E, D is not at the end. B is sitting immediate right of E, F is not at the right of end, but D is sitting 3rd left of E. Which of the following is right of D.

(a) A (b) F (c) E (d) C

Explanation: Answer (a)

According to the question A, B, C, D, E and F are sitting as follows.



Clearly A is sitting to the right of D.

2. Read the following information carefully and then answer the questions (i), (ii) and (iii).

Six friends A, B, C, D, E and F are sitting on a bench, facing towards North.

- I. A is sitting next to B.
- II. C is sitting left to D.
- III. D is not sitting with E.
- IV. E is on the left end of the bench.
- V. C is third position from right.
- VI. A is on the right side of B and to the right side of E.
- VII. A and C are sitting together.
- VIII. F is sitting Right of D.

(i) At what position A is sitting?

- (a) Between B and C (b) Between D and C
(c) Between E and D (d) Between C and E

(ii) What is position of B?

- (a) Second from right (b) Centre
(c) Extreme left (d) Second from left

(iii) What is position of D?

- (a) Extreme from left (b) Extreme right
(c) Third from left (d) Second from right.

Explanation: Arrangement according to the question is as follows.



2(i) Answer (a), A is sitting between B and C

2(ii) Answer (d), B is sitting second from left.

2 (iii) Answer (d), D is second from right

3. Read the following information carefully to answer the questions (i) and (ii)

- I. P, Q, R, S, T, U, and V are sitting along a circle facing the centre
- II. P is between V and S
- III. R, who is second to the right of S, is between Q and U
- IV. Q is not neighbour of T

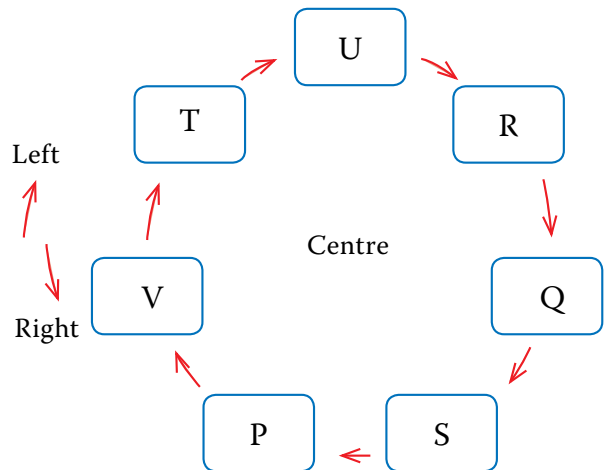
- 3 (i) Which of the following statement is a correct statement?

- (a) V is between T and P (b) S is second to left of V
(c) R is third to the left of P (d) P is to the immediate right of S

(ii) What is the position of P ?

- (a) P is immediate left of S (b) to the immediate left of V
(c) 2nd to the left of R (d) 2nd to the right of Q

Explanation: Following seating arrangement is formed from the given information.



(i) Answer (a), based on diagram V is sitting T and P.

(ii) Answer (a), based on diagram P is immediate left of S.

4. Read the following information carefully to answer the questions given below:

Seven boys A, B, C, D, E, F and G are standing in a line

- I. G is between A and D
- II. F and A have one boy between them
- III. D and C have two boys between them.
- IV E is immediate right of F.
- V. C and B have three boys between them

(i) Who is second from right?

- (a) C (b) G (c) E (d) F

(ii) Who is standing in the centre?

- (a) A (b) D (c) C (d) G

Explanation: Arrangements according to the question.



(i) Answer (d), Clearly F is second from right.

(ii) Answer (a), Clearly A is standing in the centre.

FOUNDATION: PAPER 3 LOGICAL REASONING CHAPTER 12. BLOOD RELATIONS

Blood relations of a group of persons are given in jumbled form. In these tests, the questions which are asked depend on relation.

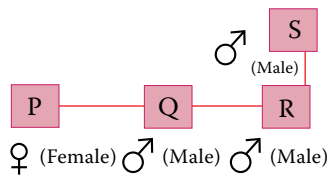
1. P is the sister of Q, Q is the brother of R, R is son of S. How S is related to P?

- (a) Father
(b) Daughter
(c) Son
(d) Uncle

Explanation:

Answer: (a)

Based on the diagram Q and R brothers and P is their sister. Therefore, S is the father P.



2. A and B are brothers, C is A's mother, D is C's father, E is B's son. How is B related to D.

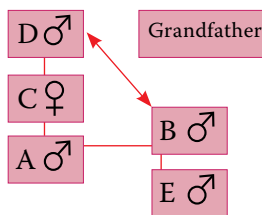
- (a) Son (b) Grandfather
(c) Grandson
(d) Great Grandfather

Explanation: Answer(c)

D is father of C.

C is mother of A and B.

Therefore, B is Grandson of D.



3. A man showed a boy next to him and said – "he is the son of my wife's sister-in-law, but I am the only child of my parents". How is my son related to him?

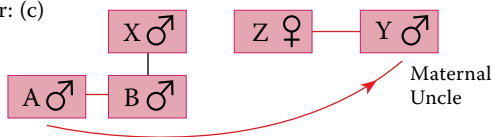
- (a) Nephew (b) Cousin (c) Brother (d) Uncle

Explanation: Answer (b). The boy is the son of man's brother-in-law. Therefore, man's son is the cousin of that boy.

4. A and B are brothers. X is the father of B, Z is the only sister of Y and Y is maternal uncle of A, what is Z related to X?

- (a) Sister (b) Brother (c) Wife (d) Mother

Answer: (c)



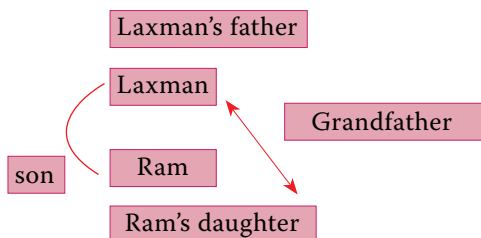
Based on the diagram A and B are brothers.

Y is brother of Z, therefore z is a female and Z is wife of X.

5. Introducing Ram to guests, Laxman said, "His father is the only son of my father". How is Ram's daughter related Laxman?

- (a) Nephew (b) Grandson (c) Grandfather (d) Son

Explanation Answer: (c)



Only son of Laxman's father is Laxman himself. Therefore, Ram's father is Laxman.

Therefore, Laxman is Grandfather of Ram's daughter.

6. Pointing to a man in photograph, a woman said "His brother's father is the only son of my grandfather". Then How is women related to the man's son in the photograph?

- (a) Daughter (b) Mother (c) Aunt (d) Sister

Explanation: Answer: (c) Only son of woman's grandfather means father of that woman.

Father of women is the father of man's brother and hence father of that man.

Therefore, the women is sister of the man and aunt to his son in photograph.

7. Read the following information carefully and answer the questions that follow

I. 'A + B' means A is the son of B.

II. 'A - B' means A is the wife of B.

III. 'A × B' means A is the sister of B.

IV. 'A ÷ B' means A is the mother of B.

V. 'A \$ B' means A is the brother of B.

(i) What does P + R - Q means

- (a) Q is the father of P (b) Q is the son P

- (c) Q is the uncle of P (d) Q is the brother of P.

(ii) What does P × R ÷ Q meaning

- (a) P is the brother of R (b) P is the father of Q

- (c) P is the aunt of Q (d) P is the nephew of Q

(iii) What does P \$ R + Q mean?

- (a) P is the aunt of Q (b) P is the son of Q

- (c) P is the niece of Q (d) P is the sister of Q

(iv) What does P \$ R ÷ Q mean?

- (a) P is the aunt of Q (b) P is the sister of Q

- (c) Q is the niece of P (d) P is the uncle of Q

Explanation:

(i) Option (a), P + R - Q, means P is the son of R, R is wife of Q, So Q is the father of P.

(ii) Option (c), P × R ÷ Q, means P is the sister of R, R is the mother of Q, So P is the aunt of Q.

(iii) Option (b), P \$ R + Q, means P is the brother of R, R is the son of Q, So P is the son of Q.

(iv) Option (d), P \$ R ÷ Q means P is the brother of R, R is the mother of Q. So P is uncle of Q.

8. On the basis of this information, you have to select the option which shows that A is the grandfather of T.

I. 'S x T' means that S is the mother of T,

II. 'S + T' means that S is the father of T,

III. 'S ÷ T' means that S is the brother of T.

- (a) A + S + B ÷ T

- (b) A x B + C ÷ T

- (c) A + C ÷ T

- (d) A + B ÷ C x T

Explanation:

Option (a) represents that A is the grandfather of T

(i) B ÷ T => B is the brother of T.

(ii) S + B => S is the father of B, hence S will be father of T [from information (i)].

(iii) A + S => A is the father of S, hence A will be grandfather of B and hence A is the grandfather of T.

CHAPTER.13 SYLLOGISM

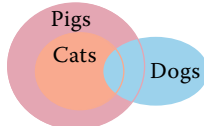
Syllogism is a 'Greek' word that means inference or deduction. As such inferences are based on logic, then these inferences are called logical deduction. These deductions are based on propositions (premise). In this section we are going to understand the few basic important questions. These deductions are based on propositions (premise).

'Syllogism' checks basic aptitude and ability of a candidate to derive inferences from given statements using step by step methods of solving problems.

Directions: In each group of questions below are two or more statements followed by two or more conclusions. You have to take the given statements to be true even if they seem to be variance from commonly known facts. Read the conclusion and then decide which of the conclusions logically follows from given statements, disregarding commonly known facts.

1.
 Statements: I. Some dogs are cats. II. All cats are pigs.
 Conclusions: I. Some cats are dogs. II. Some dogs are pigs.

- Given answer:
 (a) If only conclusion I follows
 (b) If only conclusion II follows
 (c) If either conclusion I or II follows
 (d) If neither conclusion I nor II follows
 (e) If both I and II follow
 Answer: (e), Conclusion I and II follows



Some dogs are cats (I-Type), All cats are pigs (A-type)
 $I + A = I$ type conclusion = Some dogs are pigs = Conclusion II
 Again, some dogs are cats \rightarrow (Conversion) Some cats are dogs = Conclusion I
 Clearly both the conclusion I and II follows.

2.
 Statements: I. Some cats are dogs. II. Some dogs are stones.
 Conclusions: I. No cat is stone. II. All dogs are stones. III. Some stones are cats. IV. No dog is cat.

- Given answer:
 (a) only conclusion I and III follow
 (b) only conclusion II and III follow.
 (c) only I, III and IV follow
 (d) none follows
 Answer: (d)

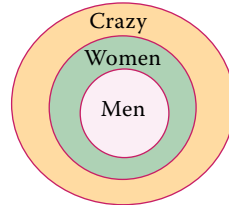
Explanation: Both the premises are particular Affirmative (I-type). No conclusion follows from the two particular premises. Conclusions I and III from complementary pair. Therefore, either conclusion I or III follows.



3.
 Statements: I. All men are women. II. All women are crazy.
 Conclusions: I. All men are crazy. II. All the crazy are men. III. Some of the crazy are men. IV. Some of the crazy are women.

- Answers
 (a) None of the conclusions follows
 (b) All the conclusions follow
 (c) Only I, III, and IV follow
 (d) Only II and III follow

Answer: (c), Only, I, III, and IV follow.
 Explanation: Venn diagram:



Both the premises are universal Affirmative (A-type)
 All men are women \leftrightarrow All women are crazy.
 $A + A \rightarrow A$ -type of conclusion.
 "All men are crazy".
 This is conclusion I.
 Conclusion III is the converse of it.
 Conclusion IV is the converse of Statement II.

4.
 Statements: I. No colour is a paint. II. No paint is a brush.
 Conclusions: I. No colour is a brush. II. All brushes are colours.

- Given answer:
 (a) If only conclusion I follows.
 (b) If only conclusion II follows.
 (c) If either conclusion I or II follows.
 (d) If neither conclusion I nor II follows.
 (e) both conclusion I and II follows.
 Answer: (d) Venn diagram



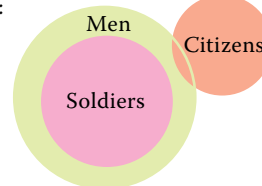
Conclusions: I No colour is a paint (X). II. All brushes are colours (X) Possible diagram as follows



5.
 Statements: I. All soldiers are men. II. Some citizens are soldiers.
 Conclusions: I. Some citizens are men. II. All soldiers are citizens.

Answers
 (a) Only I follows.
 (b) Neither I nor II follows
 (c) Only II follows
 (d) Only I and II follow
 Answer: (a) We can align the premises by changing their orders. Some citizens are soldiers.

All soldiers are men. We know that
 $I + A \rightarrow I$ Type conclusion.
 Hence our conclusion would be "some citizens would be men."
 Venn diagram:



BUSINESS MATHEMATICS AND LOGICAL REASONING & STATISTICS

CAPSULE: FOUNDATION PAPER 3: BUSINESS MATHEMATICS LOGICAL REASONING AND STATISTICS: CHAPTER 15 UNIT-I: MEASURES OF CENTRAL TENDENCY

At the foundation level with regards to Paper 3 Statistics part of the topic Measures of Central Tendency is very important for students not only to acquire professional knowledge but also for examination point of view. Here in this capsule an attempt is made for solving and understanding the concepts of Measures of Central Tendency with the help of following questions with solutions.

Definition of Central Tendency: Central tendency defined as the tendency of a given set of observations to cluster around a single central or middle value and the single value that represents the given set of observations is described as a measure of central tendency or, location, or average.

Following are the different measures of central tendency:

- Arithmetic Mean (AM)
- Median (Me)
- Mode (Mo)
- Geometric Mean (GM)
- Harmonic Mean (HM)

Criteria for an Ideal Measure of Central Tendency

- It should be properly and unambiguously defined.
- It should be easy to comprehend.
- It should be simple to compute.
- It should be based on all the observations.
- It should have certain desirable mathematical properties.
- It should be least affected by the presence of extreme observations.

Arithmetic Mean: defined as the sum of all the observations divided by the number of observations. Thus, if a variable x assumes n values $x_1, x_2, x_3, \dots, x_n$, then the AM of x , to be denoted by, $\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$, $\bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$ is given by, $\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$, $\bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$

In case of a simple frequency distribution relating to an attribute, we have

$$\bar{x} = \frac{\sum_{i=1}^n f_i x_i}{\sum_{i=1}^n f_i}$$

$$\bar{x} = \frac{\sum_{i=1}^n f_i x_i}{N} \text{ assuming the observation } x_i \text{ occurs } f_i \text{ times, } i=1,2,3,\dots, n \text{ and } N \leq n.$$

In case of grouped frequency distribution also we may use formula with x_i as the mid value of the i -th class interval, on the assumption that all the values belonging to the i -th class interval are equal to x_i .

If classification is uniform, we consider the following formula for the computation of AM from grouped frequency distribution:

$$\bar{x} = A + \frac{\sum f_i d_i}{N} \times C$$

Where, $d_i = \frac{x_i - A}{C}$ A = Assumed Mean C = Class Length

• Properties of AM

- If all the observations assumed by a variable are constants, say k , then the AM is also k .
- The algebraic sum of deviations of a set of observations from their AM is zero
- i.e. for unclassified data, $\sum (x_i - \bar{x}) = 0$ and for grouped frequency distribution, $\sum (f_i (x_i - \bar{x})) = 0$
- AM is affected due to a change of origin and/or scale which implies that if the original variable x is changed to another variable y by effecting a change of origin, say a , and scale say b , of x i.e. $\bar{y} = a + b\bar{x}$, then the AM of y is given by
- If there are two groups containing n_1 and n_2 observations and \bar{x}_1 and \bar{x}_2 as the respective arithmetic means, then combined AM is given by $\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$

Question 1: Following are the daily wages in rupees of a sample of 10 workers: 58, 62, 48, 53, 70, 52, 60, 84, 75, 100. Compute the mean wage.

Solution: Let x denote the daily wage in rupees.

Applying the mean wage is given by,

$$\bar{x} = \frac{\sum_{i=1}^{10} x_i}{10} = \frac{58 + 62 + 48 + 53 + 70 + 52 + 60 + 84 + 75 + 100}{10} = \frac{\text{₹}662}{10} = \text{₹}66.2$$

Question 2: Compute the mean weight of a group of B. Com students of Sri Ram College from the following data:

Weight in kgs	44-48	49-53	54-58	59-63	64-68	69-73
No. of students	3	4	5	7	9	12

Solution: Computation of mean weight of 40 B. Com students Applying formula, we get the average weight as

$$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{2495}{40} \text{ kgs.} = 62.38 \text{ kgs.}$$

Weight in kgs. (1)	No. of Student (f _i) (2)	Mid-Value (x _i) (3)	f _i x _i (4) = (2) x (3)
44 - 48	3	46	138
49 - 53	4	51	204
54 - 58	5	56	280
59 - 63	7	61	427
64 - 68	9	66	594
69 - 73	12	71	852
Total	40	-	2495

Question 3: Find the AM for the following distribution:

Class Interval	5-14	15-24	25-34	35-44	45-54	55-64
Frequency	10	18	32	26	14	10

Solution: Any mid value can be taken as A. However, usually A is taken as the middle most mid-value for an odd number of class intervals and any one of the two middle most mid-values for an even number of class intervals. The class length is taken as C.

The required AM is given by

$$\bar{x} = A + \frac{\sum f_i d_i}{N} \times C = 39.5 + \frac{-64}{110} \times 10 = 39.5 - 5.82 = 33.68$$

Table: Computation of AM

Class Interval	Frequency (f _i)	Mid-Value (x _i)	d _i = $\frac{x_i - A}{C}$	f _i d _i
(1)	(2)	(3)	(4)	(5) = (2) x (4)
5-14	10	9.5	-3	-30
15-24	18	19.5	-2	-36
25-34	32	29.5	-1	-32
35-44	26	39.5(A)	0	0
45-54	14	49.5	1	14
55-64	10	59.5	2	20
Total	110	-	-	-64

BUSINESS MATHEMATICS AND LOGICAL REASONING & STATISTICS

Question 4: Given that the mean height of a group of students is 67.45 inches. Find the missing frequencies for the following incomplete distribution of height of 100 students.

Height in inches	60-62	63-65	66-68	69-71	72-74
No. of students	5	18	-	-	8

Solution: Let x denote the height and f_3 and f_4 as the two missing frequencies

Table: Estimation of missing frequencies

Class Interval	Frequency	Mid-Value (x_i)	$d_i = \frac{x_i - A}{C} = \frac{x_i - 67}{3}$	$F_i d_i$ (f_i)
(1)	(2)	(3)	(4)	(5)
				$= (2) \times (4)$
60-62	5	61	-2	-10
63-65	18	64	-1	-18
66-68	f_3	67 (A)	0	0
69-71	f_4	70	1	f_4
72-74	8	73	2	16
Total	$31 + f_3 + f_4$	-	-	$-12 + f_4$

As given, we have

$$31 + f_3 + f_4 = 100, f_3 + f_4 = 69 \dots \dots \dots (1)$$

$$\bar{x} = 67.45$$

and $A + \frac{\sum f_i d_i}{N} \times C = 67.45 = 67 + \frac{(12 + f_4)}{100} \times 3 = 67.45$

$$(-12 + f_4) \times 3 = (67.45 - 67) \times 100$$

$$-12 + f_4 = 15, f_4 = 27$$

On substituting 27 for f_4 in (1), we get $f_3 + 27 = 69, f_3 = 42$. Thus, the missing frequencies would be 42 and 27.

Question 5: The mean salary for a group of 40 female workers is ₹5,400 per month and that for a group of 60 male workers is ₹7,800 per month. What is the combined mean salary?

Solution: As given $n_1 = 40, n_2 = 60, \bar{x}_1 = ₹5,400$ and $\bar{x}_2 = ₹7,800$ hence, the combined mean salary per month is

$$\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2} = \frac{40 \times ₹5,400 + 60 \times ₹7,800}{40 + 60} = ₹6,840.$$

Question 6: The mean weight of 150 students (boys and girls) in a class is 60 kg. The mean weight of boy student is 70 kg and that of girl student is 55 kg. Find number of boys and girls in that class.

Solution: Let the number of boy students be n_1 and girl students be n_2 , as given $n_1 + n_2 = 150$,

Then $n_2 = 150 - n_1$, also $\bar{x} = 60, \bar{x}_1 = 70, \bar{x}_2 = 55$

$$\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}, 60 = \frac{n_1 \times 70 + (150 - n_1) \times 55}{150}$$

$$60 = \frac{70n_1 + 8250 - 55n_1}{150} = \frac{15n_1 + 8250}{150}$$

$$= 9000 = 15n_1 + 8250$$

$$15n_1 = 750, n_1 = 50$$

$$n_2 = 150 - n_1 = 150 - 50 = 100$$

Therefore, number of boys (n_1) = 50.

Number of girls (n_2) = 100

Question 7: The average salary of a group of unskilled workers is Rs. 10,000 and that of a group of skilled workers is Rs. 15000. If the combined salary is Rs. 12000, then what is the percentage of skilled workers

Solution: Let x be unskilled and y be skilled
 $10000x + 15000y = 12000(x+y) = 12000x + 12000y$
 $2000x = 3000y$ then $2x = 3y$
 skilled workers is $2x/3$
 total workers $x + 2x/3 = 5x/3$
 percentage of skilled = $2x/3$ divided by $5x/3 = 40\%$

Question 8: The average age of a group of 10 students was 20 years. The average age increased by two years when the two new students joined in the group. What is the average age of two new students joined who joined in the group?

Solution: Average age of 10 students = 20 years, then sum of ages of 10 students = 200 years
 If the two boys are included, then total number of students = $10 + 2 = 12$
 And average increased by two years = $20 + 2 = 22$
 The average age of 12 students = 22, then sum of ages of 12 students = $22 \times 12 = 264$
 The Sum of ages of two boys = $264 - 200 = 64$
 Average age of boys = $64/2 = 32$

Median

- Partitioned Values
- As compared to AM, median is a positional average which means that the value of the median is dependent upon the position of the given set of observations for which the median is wanted. Median, for a given set of observations, may be defined as the middle-most value when the observations are arranged either in an ascending order or a descending order of magnitude.

Question 9: The median of the data 13, 8, 11, 6, 4, 15, 2, 18, 20 is **Solution:** Arranging the data in an ascending order, we get 2, 4, 6, 8, 11, 13, 15, 18, 20

Here $n = 9$, which is odd number of observations.

$$\text{Median} = \left(\frac{n+1}{2} \right)^{\text{th}} \text{ item} = \left(\frac{9+1}{2} \right)^{\text{th}} \text{ item} = 5^{\text{th}} \text{ item} = 11$$

Question 10: What is the median for the observations 5, 8, 6, 9, 11 and 4

Solution: We write in ascending order 4, 5, 6, 8, 9 and 11

Here $n = 6$. So Median = Average of 3rd and 4th term = $\frac{6+8}{2} = 7$

In case of a grouped frequency distribution, we find median from the cumulative frequency distribution of the variable under consideration.

$$M = l_1 + \left(\frac{\frac{N}{2} - N_1}{N_u - N_1} \right) \times C$$

Where,

l_1 = lower class boundary of the median class i.e. the class containing median.

N = total frequency.

N_1 = less than cumulative frequency corresponding to l_1 . (Pre median class)

N_u = less than cumulative frequency corresponding to l_2 . (Post median class)

l_2 being the upper class boundary of the median class.

$C = l_2 - l_1$ = length of the median class.

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Question 11: What is the Median for the following data?

Marks	5-14	15-24	25-34	35-44	45-5	50-59
No. of Students	10	18	32	26	14	10

Solution: First, we find the cumulative frequency distribution which is exhibited in the table

Marks	Frequency (f)	Marks	Less than Cumulative Frequency (CF)
5-14	10	14.5	10
15-24	18	24.5	28(N _l)
25-34	32	34.5	60(N _u)
35-44	26	44.5	86
45-54	14	54.5	100
55-64	10	64.5	110
Total (N)	110		

We find from table $N/2 = 55$

$\frac{N}{2} = \frac{110}{2} = 55$ lies between the two cumulative frequencies 28 and 60 i.e. $28 < 55 < 60$.

Thus, we have $N_l = 28$, $N_u = 60$, $l_1 = 24.5$ and $l_2 = 34.5$.

Hence $C = 34.5 - 24.5 = 10$.

Substituting these values formula, we get,

$$M = 24.5 + \frac{55 - 28}{60 - 28} \times 10 = 24.5 + 8.44 = 32.94$$

Question 12: Find the missing frequency from the following data, given that the median mark is 23.

Marks	0-10	10-20	20-30	30-40	40-50
No. of Students	5	8	?	6	3

Solution: Let us denote the missing frequency by f_3 . Following table shows the relevant computation.

Table (Estimation of Missing frequency)	
Marks	Less than cumulative frequency
0	0
10	5
20 (l_1)	13(N_l)
30 (l_2)	13 + f_3 (N_u)
40	19 + f_3
50	22 + f_3

Going through the mark column, we find that $20 < 23 < 30$. Hence $l_1 = 20$, $l_2 = 30$ and accordingly $N_l = 13$, $N_u = 13 + f_3$. Also the total frequency i.e. N is $22 + f_3$. Thus,

$$M = l_1 + \left(\frac{N - N_l}{N_u - N_l} \right) \times C$$

$$23 = 20 + \left(\frac{22 + f_3}{13 + f_3} \right) - 13 \times 10$$

$$3 = \frac{22 + f_3 - 26}{f_3} \times 5, \quad 3f_3 = 5f_3 - 20, \quad f_3 = 20$$

$f_3 = 10$, So, the missing frequency is 10.

Properties of median: We cannot treat median mathematically; the way we can do with arithmetic mean. We consider below two important features of median.

(i) If x and y are two variables, to be related by $y = a + bx$ for any two constants a and b , then the median of y is given by

$$y_{me} = a + bx_{me}$$

For example, if the relationship between x and y is given by $2x - 5y = 10$ and if x_{me} i.e. the median of x is known to be 16.

$$\text{Then } 2x - 5y = 10$$

$$\Rightarrow y = -2 + 0.40x$$

$$\Rightarrow y_{me} = -2 + 0.40x_{me}$$

$$\Rightarrow y_{me} = -2 + 0.40 \times 16$$

$$\Rightarrow y_{me} = 4.40$$

(ii) For a set of observations, the sum of absolute deviations is minimum when the deviations are taken from the median. This property states that $\sum |x_i - A|$ is minimum if we choose A as the median.

PARTITION VALUES OR QUANTILES :

These may be defined as values dividing a given set of observations into a number of equal parts. When we want to divide the given set of observations into two equal parts, we consider median. Similarly, quartiles are values dividing a given set of observations into four equal parts. So there are three quartiles – first quartile or lower quartile denoted by Q_1 , second quartile or median to be denoted by Q_2 or Me and third quartile or upper quartile denoted by Q_3 . First quartile is the value for which one fourth of the observations are less than or equal to Q_1 and the remaining three – fourths observations are more than or equal to Q_1 . In a similar manner, we may define Q_2 and Q_3 .

Deciles :

Deciles are the values dividing a given set of observation into ten equal parts. Thus, there are nine deciles to be denoted by $D_1, D_2, D_3, \dots, D_9$. D_1 is the value for which one-tenth of the given observations are less than or equal to D_1 and the remaining nine-tenth observations are greater than or equal to D_1 when the observations are arranged in an ascending order of magnitude.

percentiles or centiles

Percentiles divide a given set of observations into 100 equal parts. The points of sub-divisions being P_1, P_2, \dots, P_{99} . P_1 is the value for which one hundredth of the observations are less than or equal to P_1 and the remaining ninety-nine hundredths observations are greater than or equal to P_1 once the observations are arranged in an ascending order of magnitude.

For unclassified data, the p^{th} quartile is given by the $(n+1)p^{\text{th}}$ value, where n denotes the total number of observations. $p = 1/4, 2/4, 3/4$ for Q_1, Q_2 and Q_3 respectively. $p = 1/10, 2/10, \dots, 9/10$.

For D_1, D_2, \dots, D_9 respectively and lastly $p = 1/100, 2/100, \dots, 99/100$ for $P_1, P_2, P_3, \dots, P_{99}$ respectively.

In case of a grouped frequency distribution, we consider the following formula for the computation of quartiles.

$$Q = l_1 + \left(\frac{N_p - N_l}{N_u - N_l} \right) \times C$$

The symbols, except p , have their usual interpretation which we have already discussed while computing median and just like the unclassified data, we assign different values to p depending on the quartile.

Another way to find quartiles for a grouped frequency distribution is to draw the ogive (less than type) for the given distribution. In order to find a particular quartile, we draw a line parallel to the horizontal axis through the point N_p . We draw perpendicular from the point of intersection of this parallel line and the ogive. The x -value of this perpendicular line gives us the value of the quartile.

Question 13: Following are the wages of the labourers: ₹ 82, ₹ 56, ₹ 90, ₹ 50, ₹ 120, ₹ 75, ₹ 75, ₹ 80, ₹ 130, ₹ 65. Find Q_1, D_6 and P_{82} .

Solution: Arranging the wages in an ascending order, we get ₹ 50, ₹ 56, ₹ 65, ₹ 75, ₹ 75, ₹ 80, ₹ 82, ₹ 90, ₹ 120, ₹ 130. Hence, we have

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$$Q_1 = \frac{(n+1)}{4} \text{th value} = \frac{(10+1)}{4} \text{th value} = 2.75^{\text{th}} \text{ value}$$

$$= 2^{\text{nd}} \text{ value} + 0.75 \times \text{difference between the third and the } 2^{\text{nd}} \text{ values.}$$

$$= ₹ [56 + 0.75 \times (65 - 56)] = ₹ 62.75$$

$$D_6 = (10 + 1) \times \frac{6}{10} \text{th value} = 6.60^{\text{th}} \text{ value}$$

$$= 6^{\text{th}} \text{ value} + 0.60 \times \text{difference between the } 7^{\text{th}} \text{ and the } 6^{\text{th}} \text{ values.}$$

$$= ₹ (80 + 0.60 \times 2) = ₹ 81.20$$

$$P_{82} = (10+1) \times \frac{82}{100} \text{th value} = 9.02^{\text{th}} \text{ value}$$

$$= 9^{\text{th}} \text{ value} + 0.02 \times \text{difference between the } 10^{\text{th}} \text{ and the } 9^{\text{th}} \text{ values}$$

$$= ₹ (120 + 0.02 \times 10) = ₹ 120.20$$

Question 14: Compute the Third Quartile and 65th percentile for the following data

Profits '000 Rs	Less than 10	10-19	20-29	30-39	40-49	50-59
No. of firms	5	18	38	20	9	2

Solution:

Profits'000(Rs.)	Frequency (f)	Cumulative Frequency (CF)
Less than 9.5	5	5
9.5-19.5	18	23
19.5-29.5	38	61
29.5-39.5	20	81
39.5-49.5	9	90
49.5-59.5	2	92
110		

$$Q_3 = \text{Third Quartile} = \frac{3N}{4} = \frac{3 \times 92}{4} = 69 \quad Q_3 \text{ lies } 29.5 \text{ and } 39.5$$

$$Q_3 = 29.5 + \left(\frac{69 - 61}{20} \right) \times 10 = 33.5$$

$$Q_3 = ₹ 33,500$$

$$\text{For } 65^{\text{th}} \text{ percentile} = P_{65} = \frac{iN}{100} = \frac{65 \times 92}{100} = 59.8 = 65^{\text{th}}$$

percentile lies in the class 19.5-29.5, here $l = 19.5$, $c = 23$, $f = 38$ and $C = 10$

$$P_{65} = 19.5 + \frac{(59.8 - 23)}{38} \times 10$$

$$P_{65} = 29.184$$

$$= ₹ 29,184$$

Question 15: Compute mode for the distribution for the following distribution

Class Interval	350-369	370-389	390-409	410-429	430-449	450-469
Frequency	15	27 (f ₋₁)	31 (f ₀)	19 (f ₁)	13	6

Solution: Going through the frequency column, we note that the highest frequency i.e., $f_0 = 31$ and $f_{-1} = 27$, $f_1 = 19$, $LCB = 389.5$, $C = 409.5 - 389.5 = 20$

$$\text{Mode} = 389.5 + \frac{(31 - 27)}{2 \times 31 - (27 + 19)} \times 20$$

$$\text{Mode} = 389.5 + \frac{4}{16} \times 20 = 389.5 + 5 = 394.5$$

Question 16: For a moderately skewed distribution of marks in statistics for a group of 200 students, the mean mark and median mark were found to be 55.60 and 52.40. What is the modal mark?

Solution: Since in this case, mean = 55.60 and median = 52.40, applying, we get the modal mark as,
 Mode = $3 \times \text{Median} - 2 \times \text{Mean} = 3 \times 52.40 - 2 \times 55.60 = 46$.

Question 17: If x and y related by $x - y - 10 = 0$ and mode of x is known to be 23, then the mode of y is :

Solution: Mode of $x = 23$, $x - y - 10 = 0$ then $y = x - 10$, Mode of $y = 23 - 10 = 13$

Geometric Mean: For a given set of n positive observations, the geometric mean is defined as the n -th root of the product of the observations. Thus if a variable x assumes n values $x_1, x_2, x_3, \dots, x_n$, all the values being positive, then the GM of x is given by $G = (x_1 \times x_2 \times x_3 \dots \times x_n)^{1/n}$

For a grouped frequency distribution, the GM is given by $G = (x_1^f \times x_2^f \times x_3^f \dots \times x_n^f)^{1/N}$, Where $N = \sum f_i$

In connection with GM, we may note the following properties:

- Logarithm of G for a set of observations is the AM of the logarithm of the observations
- if all the observations assumed by a variable are constants, say $K > 0$, then the GM of the observations is also K .
- GM of the product of two variables is the product of their GM's i.e. if $z = xy$, then
 $GM \text{ of } z = (GM \text{ of } x) \times (GM \text{ of } y)$
- GM of the ratio of two variables is the ratio of the GM's of the two variables i.e. if $z = x/y$ then

$$GM \text{ of } z = \frac{GM \text{ of } x}{GM \text{ of } y}$$

Question 18: Find the GM of 8, 24 and 40.

Solution: As given $x_1 = 8$, $x_2 = 24$, $x_3 = 40$ and $n = 3$.
 Applying, we have $G = (8 \times 24 \times 40)^{1/3} = 8\sqrt[3]{15}$

Question 19: If GM of x is 10, and GM of y is 15, then GM of xy

Solution: According to the GM of $XY = GM \text{ of } x \times GM \text{ of } y = 10 \times 15 = 150$

Harmonic Mean: For a given set of non-zero observations, harmonic mean is defined as the reciprocal of the AM of the reciprocals of the observation. So, if a variable x assumes n non-zero values $x_1, x_2, x_3, \dots, x_n$, then the HM of x is given by

$$H = \frac{n}{\sum (1/x_i)}$$

For a grouped frequency distribution, we have $H = \frac{N}{\sum \left[\frac{f}{x_i} \right]}$

Properties of HM

- If all the observations taken by a variable are constants, say k , then the HM of the observations is also k .
- If there are two groups with n_1 and n_2 observations and H_1 and H_2 as respective HM's than the combined HM is given

$$\text{by} = \frac{\frac{n_1 + n_2}{\frac{n_1}{H_1} + \frac{n_2}{H_2}}}$$

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Question 20: A man travels at a speed of 20 km/hr and then returns at a speed of 30 km/hr. His average of the whole journey is

Solution: Harmonic Mean is the method which is preferred for the computation of average speed

$$HM = \frac{2ab}{a+b} = \frac{2 \times 20 \times 30}{20+30} = 24 \text{ km/hr}$$

Question 21: Find the HM for 4, 6 and 10.

Solution: Applying formula, we have

$$H = \frac{3}{\frac{1}{4} + \frac{1}{6} + \frac{1}{10}} = \frac{3}{0.25 + 0.17 + 0.10} = 5.77$$

Question 22: An aeroplane flies from A to B at the rate of 500 km/hr and comes back B to A at the rate of 700 km/hr. The average speed of the aeroplane is;

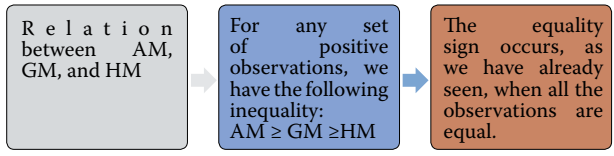
Solution: Required average speed of the aeroplane =

$$\frac{2}{\left(\frac{1}{500} + \frac{1}{700}\right)} = \frac{2 \times 3500}{7+5} = 583.33 \text{ km/hr}$$

Question 23: Find the HM for the following data:

x	2	4	8	16
f	2	3	3	2

Solution: Using formula, we get $H = \frac{10}{\frac{2}{2} + \frac{3}{4} + \frac{3}{8} + \frac{2}{16}} = 4.44$



Question 24: compute AM, GM, and HM for the numbers 6, 8, 12, 36.

Solution: In accordance with the definition, we have

$$AM = \frac{6+8+12+36}{4} = 15.50$$

$$GM = (6 \times 8 \times 12 \times 36)^{1/4} = (2^8 \times 3^4)^{1/4} = 12$$

$$HM = \frac{4}{\frac{1}{6} + \frac{1}{8} + \frac{1}{12} + \frac{1}{36}} = 9.93$$

The computed values of AM, GM, and HM establish $AM \geq GM \geq HM$

Question 25: If there are two groups with 75 and 65 as harmonic means and containing 15 and 13 observations, then the combined HM is given by

$$\text{Solution: Combined HM is given by} = \frac{\frac{n_1 + n_2}{H_1 + H_2}}{\left(\frac{15}{75} + \frac{13}{65}\right)} = 70$$

Weighted average
When the observations under consideration have a hierarchical order of importance, we take recourse to computing weighted average, which could be either weighted AM or weighted GM or weighted HM.

Weighted AM = $\frac{\sum w_i x_i}{\sum w_i}$

Weighted GM = $\text{Ante log} \left(\frac{\sum w_i \log x_i}{\sum w_i} \right)$

Weighted HM = $\frac{\sum w_i}{\sum \left(\frac{w_i}{x_i} \right)}$

FOUNDATION: PAPER 3 - BUSINESS MATHEMATICS LOGICAL REASONING AND STATISTICS

At the foundation level with regards to Paper 3 Statistics part of the topic Measures of Dispersion is very important for students not only to acquire professional knowledge but also for examination point of view. Here in this capsule an attempt is made for solving and understanding the concepts of Measures of Dispersion with the help of following questions with solutions

CHAPTER 15 UNIT-II: MEASURES OF DISPERSION

The second important characteristic of a distribution is given by dispersion. Two distributions may be identical in respect of its first important characteristic i.e. central tendency and yet they may differ on account of scatterness. The following figure shows a number of distributions having identical measure of central tendency and yet varying measure of scatterness. Obviously, distribution is having the maximum amount of dispersion.

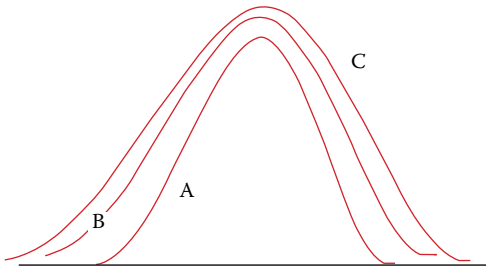


Figure: Showing distributions with identical measure of central tendency and varying amount of dispersion.

Dispersion

for a given set of observations may be defined as the amount of deviation of the observations, usually, from an appropriate measure of central tendency. Measures of dispersion may be broadly classified into the following:

Absolute measures of dispersion

- (i) Range
- (ii) Mean Deviation
- (iii) Standard Deviation
- (iv) Quartile Deviation

Relative measures of dispersion

- (i) Coefficient of range.
- (ii) Coefficient of Mean Deviation
- (iii) Coefficient of Variation
- (iv) Coefficient of Quartile Deviation

Distinction between the absolute and relative measures of dispersion :

- Absolute measures are dependent on the unit of the variable under consideration whereas the relative measures of dispersion are unit free.
- For comparing two or more distributions, relative measures and not absolute measures of dispersion are considered.
- Compared to absolute measures of dispersion, relative measures of dispersion are difficult to compute and comprehend.

Characteristics for an ideal measure of dispersion

An ideal measure of dispersion should be properly defined, easy to comprehend, simple to compute, based on all the observations, unaffected by sampling fluctuations and amenable to some desirable mathematical treatment

- Range :
- For a given set of observations, range may be defined as the difference between the largest and smallest of observations.
 - Thus if L and S denote the largest and smallest observations respectively then $\text{Range} = L - S$

$$\text{Coefficient of Range} = \frac{L - S}{L + S} \times 100$$

For a grouped frequency distribution: Range is defined as the difference between the two extreme class boundaries. The corresponding relative measure of dispersion is given by the ratio of the difference between the two extreme class boundaries to the total of these class boundaries, expressed as a percentage.

Range remains unaffected due to a change of origin but affected in the same ratio due to a change in scale i.e., if for any two constants a and b, two variables x and y are related by $y = a + bx$, Then the range of y is given by

$$R_y = |b| \times R_x$$

Example 1: Following are the wages of 8 workers expressed in rupees: 80, 65, 90, 60, 75, 70, 72, 85. Find the range and find its coefficient.

Solution: The largest and the smallest wages are $L = ₹ 90$ and $S = ₹ 60$
Thus $\text{range} = ₹ 90 - ₹ 60 = ₹ 30$

$$\text{Coefficient of range} = \frac{90 - 60}{90 + 60} \times 100 = 20$$

Example 2: What is the range and its coefficient for the following distribution of weights?

Weight in kgs	10-19	20-29	30-39	40-49	50-59
No. of Students	11	25	16	7	3

Solution: The lowest class boundary is 9.50 kgs. and the highest-class boundary is 59.50 kgs. Thus, we have
 $\text{Range} = 59.50 \text{ kgs.} - 9.50 \text{ kgs.} = 50 \text{ kgs.}$

$$\text{Also, coefficient of range} = \frac{59.50 - 9.50}{59.50 + 9.50} \times 100 = \frac{50}{69} \times 100 = 72.46$$

Example 3: If the relationship between x and y is given by $2x + 3y = 10$ and the range of x is ₹ 15, what would be the range of y?

Solution: Since $2x + 3y = 10$

Therefore, $y = \frac{10}{3} - \frac{2}{3}x$, Applying the range of y is given by

$$R_y = |b| \times R_x = \frac{2}{3} \times ₹ 15 = ₹ 10.$$

Mean Deviation : Since range is based on only two observations, it is not regarded as an ideal measure of dispersion. A better measure of dispersion is provided by mean deviation which, unlike range, is based on all the observations. For a given set of observation, mean deviation is defined as the arithmetic mean of the absolute deviations of the observations from an appropriate measure of central tendency. Hence if a variable x assumes n values $x_1, x_2, x_3, \dots, x_n$, then the mean deviation of x about an average A is given by

$$MD_A = \frac{1}{n} \sum |x_i - A|$$

For a grouped frequency distribution, mean deviation about A is given by $MD_A = \frac{1}{n} \sum |x_i - A| \cdot f_i$

Where x_i and f_i denote the mid value and frequency of the i^{th} class interval and $N = \sum f_i$

In most cases we take A as mean or median and accordingly, we get mean deviation about mean or mean deviation about median or mode.

A relative measure of dispersion applying mean deviation is given by

$$\text{Coefficient of Mean Deviation} = \frac{\text{Mean deviation about A}}{A} \times 100$$

Mean deviation takes its minimum value when the deviations are taken from the median. Also mean deviation remains unchanged due to a change of origin but changes in the same ratio due to a change in scale i.e. if $y = a + bx$, a and b being constants, then $MD \text{ of } y = |b| \times MD$

Example 4: What is the mean deviation about mean for the following numbers?
50,60,50,50,60,60,60,50,50,50,60,60,50

Solution: The mean is given by

$$\bar{x} = \frac{50 + 60 + 50 + 50 + 60 + 60 + 60 + 50 + 50 + 50 + 60 + 60 + 50}{14} = \frac{770}{14} = 55$$

x_i	50	60	50	50	60	60	60	50	50	50	60	60	60	50	Total
$ x_i - \bar{x} $	5	5	5	5	5	5	5	5	5	5	5	5	5	5	70

Thus, mean deviation about mean is given by $\frac{\sum |x_i - \bar{x}|}{n} = \frac{70}{14} = 5$

Example. 5 : The coefficient of Mean Deviation about the first 9 natural numbers ?

Solution: The Mean of first 9 natural numbers = $\frac{n+1}{2} = \frac{9+1}{2} = 5$

coefficient of Mean Deviation about the first 9 natural numbers =

$$\frac{\text{Mean deviation about A}}{A} \times 100 = \frac{\frac{20}{5}}{5} = \frac{4}{9} \times 100 = \frac{400}{9}$$

Example. 6 : The mean deviation about the mode for the following observations 4/11, 6/11, 8/11, 9/11, ,12/11, 8/11 is

Solution: For the 4/11, 6/11, 8/11, 9/11, 12/11, 8/11 Mode is 8/11

$$\begin{aligned} \text{Mean deviation from Mode} &= \frac{\sum |x_i - \text{Mode}|}{n} \\ &= \frac{\left| \frac{4}{11} - \frac{8}{11} \right| + \left| \frac{6}{11} - \frac{8}{11} \right| + \left| \frac{8}{11} - \frac{8}{11} \right| + \left| \frac{9}{11} - \frac{8}{11} \right| + \left| \frac{12}{11} - \frac{8}{11} \right| + \left| \frac{8}{11} - \frac{8}{11} \right|}{6} \\ &= \frac{\frac{4}{11} + \frac{2}{11} + 0 + \frac{1}{11} + \frac{4}{11} + 0}{6} = \frac{\frac{11}{11}}{6} = \frac{1}{6} \end{aligned}$$

Example 7: Find mean deviations about median and the corresponding coefficient for the following profits ('000₹) of a firm during a week. 82, 56, 75, 70, 52, 80, 68.

Solution: The profits in thousand rupees is denoted by x. Arranging the values of x in an ascending order, we get 52, 56, 68, 70, 75, 80, 82.

Therefore, Median = $\left(\frac{n+1}{2}\right)^{th} = \left(\frac{7+1}{2}\right)^{th}$ item = 4th item = 70, thus, Median profit = ₹ 70,000.

Computation of Mean deviation about median

x_i	52	56	68	70	75	80	82	Total
$ x_i - Me $	18	14	2	0	5	10	12	61

Thus mean deviation about median $\frac{\sum |x_i - \text{Median}|}{n} = \frac{61 \times 1000}{7} = ₹ 8714.29$

$$\text{Coefficient of mean deviation} = \frac{\text{MD about median}}{\text{Median}} \times 100 = \frac{8714.29}{70000} \times 100 = 12.45$$

Example 8: Compute the mean deviation about the arithmetic mean for the following data:

Variable (x)	5	10	15	20	25	30
Frequency (f)	3	4	6	5	3	2

Solution: We are to apply formula as these data refer to a grouped frequency distribution the AM is given by

$$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{5 \times 3 + 10 \times 4 + 15 \times 6 + 20 \times 5 + 25 \times 3 + 30 \times 2}{3 + 4 + 6 + 5 + 3 + 2} = 16.52$$

$$\text{Mean deviation from Mean} = \frac{\sum f_i |x_i - \bar{x}|}{n} = \frac{139.56}{23} = 6.07$$

$$\text{Coefficient of MD about its AM} = \frac{\text{MD about AM}}{\text{AM}} \times 100 = \frac{6.07}{16.52} \times 100 = 36.73$$

Example 9: The mean and SD for a, b and 2 are 3 and $\frac{2}{\sqrt{3}}$ respectively, The value of ab would be

Solution: Here the mean a, b and 2 (\bar{x}) = 3, $\bar{x} = \frac{a+b+2}{3}$, $9 = a+b+2$

Then a + b = 9 - 2 = 7

$$\text{Standard deviation} = \sqrt{\frac{\sum x^2}{n} - (\bar{x})^2}$$

$$\frac{2}{\sqrt{3}} = \sqrt{\frac{\sum x^2}{n} - (3)^2}$$

$$\Rightarrow \frac{4}{3} = \frac{\sum x^2}{3} - (3)^2$$

$$\frac{4}{3} = \frac{\sum x^2}{3} - 9$$

$$\sum x^2 = 27 + 4 = 31 \Rightarrow \sum x^2 = 31$$

$$a^2 + b^2 + 2^2 = 31$$

$$a^2 + b^2 = 31 - 4 = 27$$

$$(a + b)^2 - 2ab = a^2 + b^2$$

$$(7)^2 - 2ab = 27, 2ab = 49 - 27$$

$$2ab = 22$$

$$ab = 11$$

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Example 10: If x and y are related as $4x+3y+11 = 0$ and mean deviation of x is 5.40, what is the mean deviation of y ?

Solution: Since $4x + 3y + 11 = 0$

$$\text{Therefore, } y = \left(-\frac{11}{3}\right) + \left(-\frac{4}{3}\right)x$$

$$\text{Hence MD of } y = |b| \times \text{MD of } x = \frac{4}{3} \times 5.40 = 7.20$$

Standard Deviation: Although mean deviation is an improvement over range so far as a measure of dispersion is concerned, mean deviation is difficult to compute and further more, it cannot be treated mathematically. The best measure of dispersion is, usually, standard deviation which does not possess the demerits of range and mean deviation.

Standard deviation for a given set of observations is defined as the root mean square deviation when the deviations are taken from the AM of the observations. If a variable x assumes n values $x_1, x_2, x_3, \dots, x_n$ then its standard deviation(s) is given by

$$S = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$$

For a grouped frequency distribution, the standard deviation is given by

$$S = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{N}}$$

can be simplified to the following forms for unclassified data

$$S = \sqrt{\frac{\sum x_i^2}{n} - \bar{x}^2}$$

$$= \sqrt{\frac{\sum f_i x_i^2}{N} - \bar{x}^2} \text{ for a grouped frequency distribution.}$$

Variance: The square of standard deviation, known as variance

$$\text{Variance} = s^2 = \frac{\sum x_i - \bar{x}^2}{n} \text{ for unclassified data}$$

$$= \frac{\sum f_i (x_i - \bar{x}^2)}{N} \text{ for a grouped frequency distribution}$$

Coefficient of variation (CV) = $\frac{SD}{AM} \times 100$ (A relative measure

of dispersion using standard deviation is given by Coefficient of Variation (CV) which is defined as the ratio of standard deviation to the corresponding arithmetic mean, expressed as a percentage.)

Example 11: Find the standard deviation and the coefficient of variation for the following numbers: 5, 8, 9, 2, 6

Solution: We present the computation in the following table:
Computation of standard deviation

x_i	5	8	9	2	6	$\sum x_i = 30$
x_i^2	25	64	81	4	36	$\sum x_i^2 = 210$

Applying, we get the standard deviation as

$$= \sqrt{\frac{\sum x_i^2}{n} - \bar{x}^2} = \sqrt{\frac{210}{5} - \left(\frac{30}{5}\right)^2} \quad \left(\text{since } \bar{x} = \frac{\sum x_i}{n}\right)$$

$$= \sqrt{42-36} = \sqrt{6} = 2.45$$

$$\text{The coefficient of variation is } CV = 100 \times \frac{SD}{AM} = 100 \times \frac{2.45}{6} = 40.83$$

We consider the following formula for computing standard deviation from grouped frequency distribution with a view to saving time and computational labour:

$$S = \sqrt{\frac{\sum fd_i^2}{N} - \left(\frac{\sum fd_i}{N}\right)^2} \times C, \text{ Where } d_i = \frac{x_i - A}{C}$$

Properties of standard deviation

1. If all the observations assumed by a variable are constant i.e. equal, then the SD is zero. This means that if all the values taken by a variable x is k , say, then $s = 0$. This result applies to range as well as mean deviation.

2. SD remains unaffected due to a change of origin but is affected in the same ratio due to a change of scale i.e., if there are two variables x and y related as $y = a+bx$ for any two constants a and b , then SD of y is given by $s_y = |b| s_x$

3. If there are two groups containing n_1 and n_2 observations, 1 and 2 as respective AM's, s_1 and s_2 as respective SD's, then the combined SD is given by

$$s = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}} \text{ where, } d_1 = \bar{x}_1 - \bar{x}, d_2 = \bar{x}_2 - \bar{x} \text{ and}$$

$$\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2} = \text{combined AM}$$

This result can be extended to more than 2 groups, we have

$$s = \sqrt{\frac{\sum n_i s_i^2 + \sum n_i d_i^2}{\sum n_i}} \text{ With } d = x_i - \bar{x} \text{ and } \bar{x} = \frac{\sum n_i \bar{x}_i}{\sum n_i}$$

$$\text{Where } \bar{x}_1 = \bar{x}_2 \text{ is reduced to } s = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2}}$$

4. For any two numbers a and b , standard deviation is given by $\frac{|a-b|}{2}$

$$5. \text{SD of first } n \text{ natural numbers is } SD = \frac{\sqrt{n^2 - 1}}{2}$$

Example 12: If the S.D. of x is 3, what is the variance of $(5 - 2x)$?

Solution: If $y = a + bx$, then $\sigma_y = |b| \sigma_x$

$$\text{Let } y = 5 - 2x$$

$$\therefore \sigma_y = |-2| \sigma_x$$

$$= 2 \times 3 = 6$$

$$\therefore \text{Variance } (5 - 2x) = (2)^2 \times 9 = 36$$

Example 13: The coefficient of variation of the following numbers 53, 52, 61, 60, 64, is

$$\text{Solution: } \bar{x} = \frac{(53+52+61+60+64)}{5} = 58$$

$$\therefore \sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

$$\therefore \sigma = \sqrt{\frac{(-5)^2 + (-6)^2 + 3^2 + 2^2 + 6^2}{5}} = 4.69.$$

$$\text{Coefficient of variation} = \frac{S.D.}{A.M.} \times 100$$

$$= \frac{4.69}{58} \times 100 = 8.09.$$

Example 14: What is the standard deviation of 5,5,9,9,9,10,5,10,10?

Solution: Mean = $\frac{3 \times 5 + 3 \times 9 + 3 \times 10}{9} = \frac{72}{9} = 8$.

Standard Deviation = $\sqrt{\frac{\sum f(x_i - \bar{x})^2}{n}} = \sqrt{\frac{3(9) + 3(1) + 3(4)}{9}} = \sqrt{42/9} = 2.16$.

Example 15: If x and y are related by $2x + 3y + 4 = 0$ and S.D. of x is 6, then S.D. of y is :

Solution: $y = \frac{-2}{3}x - \frac{4}{3}$;

$\sigma_y = |-2/3| \sigma_x = (2/3) \times 6 = 4$.

Example 16: If x and y are related by $y = 2x + 5$ and the S.D. and A.M. of x are known to be 5 and 10 respectively, then the coefficient of variation of y is:

Solution: $Y = 2x + 5$

$\sigma_y = |2| \sigma_x = 2 \times 5 = 10$.

Also $\bar{y} = 2\bar{x} + 5 = 20 + 5 = 25$

Coefficient of variation of y = $\frac{\sigma_y}{\bar{y}} \times 100 = \frac{10}{25} \times 100 = 40$.

Example 17: If the mean and S.D. of x are a and b respectively, then the S.D. of $\frac{x-a}{b}$ is

Solution: Let $y = \frac{(x-a)}{b} = \frac{1}{b} \cdot x - \frac{a}{b}$

$\sigma_y = \left| \frac{1}{b} \right| \sigma_x$

$\frac{1}{b} \sigma_x = 1$.

Example 18: If x and y are related by $3y = 7x - 9$ and the S.D. of y is 7, then what is the variance of x?

Solution: $3y = 7x - 9$

$x = \frac{3}{7}y + 9$

Also $\sigma_x = \left| \frac{3}{7} \right| \sigma_y = \frac{3}{7} \times 7 = 3$.

\therefore Variance : $\sigma_x^2 = 3^2 = 9$

Example 19: Which of the following companies A and B is more consistent so far as the payment of dividend is concerned ?

Dividend paid by A	5	9	6	12	15	10	8	10
Dividend paid by B:	4	8	7	15	18	9	6	6

Solution: Here $\sum x_A = 75$

$\therefore \bar{x}_A = 75/8 = 9.375$

$\sum x_A^2 = 775$

$\sigma_A^2 = \frac{\sum x_A^2}{N} - \left(\frac{\sum x_A}{N} \right)^2$

$= \frac{775}{8} - \left(\frac{75}{8} \right)^2 = 9$

$\sigma_A = 3$.

$C.V._A = \frac{\sigma_A}{\bar{x}} \times 100 = \frac{3}{9.375} \times 100 = 32$.

Also $\sum x_B = 73$, $\therefore \bar{x}_B = \frac{73}{8} = 9.125$

$\sum x_B^2 = 831$,

$\sigma_B^2 = \frac{831}{8} - \left(\frac{73}{8} \right)^2 = 20.61$

$\therefore \sigma = \sqrt{20.61} = 4.54$

$C.V._B = \frac{4.54}{9.125} \times 100 = 49.75$

$C.V_A < C.V_B$

Company A is more consistent

Example 20: Find the SD of the following distribution:

Weight(kgs)	50-52	52-54	54-56	56-58	58-60
No.of Students	17	35	28	15	5

Solution:

Weight	No. of Students	Mid - Value	$d_i = x_i - 55$	$f_i d_i$	$f_i d_i^2$
50-52	17	51	-2	-34	68
52-54	35	53	-1	-35	35
54-56	28	55	0	0	0
56-58	15	57	1	15	15
58-60	5	59	2	10	20
Total	100			-44	138

Applying, we get the SD of weight as

$= \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N} \right)^2} \times C = \sqrt{\frac{138}{100} - \frac{(-44)^2}{100}} \times 2 \text{ kgs.} = \sqrt{1.38 - 0.1936} \times 2 \text{ kgs.}$

$= 2.18 \text{ kgs}$

Example 21: The mean and variance of the 10 observations are found to be 17 and 33 respectively. Later it is found that one observation (i.e.26) is inaccurate and is removed. What is mean and standard deviation of remaining?

Solution: Mean of 10 observations = 17 then Total of the observations = $17 \times 10 = 170$

Total of the 9 observations = $170 - 26 = 144$

Changed Mean = $144/9 = 16$

Variance (σ^2) = 33

$\frac{\sum x^2}{n} - (17)^2 = 33 \Rightarrow \frac{\sum x^2}{10} = 33 + 289 = 322$

$\frac{\sum x^2}{10} = 322$

$\sum x^2 = 3220 - (26)^2 = 3220 - 676 = 2544$

Changed Variance = $\frac{\text{Changed } \sum x^2}{n} - (\text{Changed } \bar{x})^2$

$= \frac{2544}{9} - (16)^2 = 26.67$

SD of remaining observations = $\sqrt{26.67} = 5.16$

Example 22: If AM and coefficient of variation of x are 10 and 40 respectively, what is the variance of $(15-2x)$?

Solution: let $y = 15 - 2x$; AM of x = 10

Then applying formula, we get,

$s_y = 2 \times s_x$

As given $cv_x =$ coefficient of variation of x = 40 and = 10

Thus $cv_x = \frac{s_x}{\bar{x}} \times 100 \Rightarrow 40 = \frac{s_x}{10} \times 100$

$\Rightarrow s_x = 4$

Then, $S_y = 2 \times 4 = 8$ Therefore, variance of $(15-2x) = S_y^2 = 64$

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Example 23: Compute the SD of 9, 5, 8, 6, 2. Without any more computation, obtain the SD of

Sample I	-1	-5	-2	-4	-8
Sample II	90	50	80	60	20
Sample III	23	15	21	17	9

Solution:

x_i	9	5	8	6	2	30
x_i^2	81	25	64	36	4	210

The SD of the original set of observations is given by

$$s = \sqrt{\frac{210}{5} - \left(\frac{30}{5}\right)^2} = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2} = \sqrt{42 - 36} = \sqrt{6} = 2.45$$

If we denote the original observations by x and the observations of sample I by y , then we have

$$y = -10 + x$$

$$y = (-10) + (1)x$$

$$\therefore S_y = |1| \times S_x = 1 \times 2.45 = 2.45$$

In case of sample II, x and y are related as

$$Y = 10x = 0 + (10)x$$

$$\therefore S_y = |10| \times S_x$$

$$= 10 \times 2.45 = 24.50$$

And lastly, $y = (5) + (2)x$

$$\Rightarrow S_y = 2 \times 2.45 = 4.90$$

Example 24: For a group of 60 boy students, the mean and SD of stats. marks are 45 and 2 respectively. The same figures for a group of 40 girl students are 55 and 3 respectively. What is the mean and SD of marks if the two groups are pooled together?

Solution: As given $n_1 = 60$, $\bar{x}_1 = 45$, $s_1 = 2$, $n_2 = 40$, $\bar{x}_2 = 55$, $s_2 = 3$

Thus the combined mean is given by

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \frac{60 \times 45 + 40 \times 55}{60 + 40} = 49$$

$$\text{Thus } d_1 = \bar{x}_1 - \bar{x} = 45 - 49 = -4$$

$$d_2 = \bar{x}_2 - \bar{x} = 55 - 49 = 6$$

Applying formula, we get the combined SD as

$$s = \sqrt{\frac{n_1s_1^2 + n_2s_2^2 + n_1d_1^2 + n_2d_2^2}{n_1 + n_2}}$$

$$s = \sqrt{\frac{60 \times 2^2 + 40 \times 3^2 + 60 \times (-4)^2 + 40 \times 6^2}{60 + 40}}$$

$$= \sqrt{30}$$

$$= 5.48$$

Example 25: The mean and standard deviation of the salaries of the two factories are provided below:

Factory	No. of Employees	Mean Salary	SD of Salary
A	30	₹ 4800	₹ 10
B	20	₹ 5000	₹ 12

- (i) Find the combined mean salary and standard deviation of salary.
 (ii) Examine which factory has more consistent structure so far as satisfying its employees are concerned.

Solution: Here we are given

$$n_1 = 30, \bar{x}_1 = ₹ 4800, s_1 = ₹ 10,$$

$$n_2 = 20, \bar{x}_2 = ₹ 5000, s_2 = ₹ 12$$

$$(i) \text{ Combined mean} = \frac{30 \times ₹ 4800 + 20 \times 5000}{30 + 20} = ₹ 4880$$

$$d_1 = \bar{x}_1 - \bar{x} = ₹ 4,800 - ₹ 4880 = - ₹ 80$$

$$d_2 = \bar{x}_2 - \bar{x} = ₹ 5,000 - ₹ 4880 = ₹ 120$$

hence, the combined SD in rupees is given by

$$s = \sqrt{\frac{30 \times 10^2 + 20 \times 12^2 + 30 \times (-80)^2 + 20 \times 120^2}{30 + 20}} = \sqrt{9717.60} = 98.58$$

thus the combined mean salary and the combined standard deviation of salary are ₹ 4880 and ₹ 98.58 respectively.

- (ii) In order to find the more consistent structure, we compare the coefficients of variation of the two factories.

$$\text{Letting } CV_A = 100 \times \frac{S_A}{\bar{X}_A} \text{ and } CV_B = 100 \times \frac{S_B}{\bar{X}_B}$$

We would say factory A is more consistent

if $CV_A < CV_B$. Otherwise factory B would be more consistent.

$$\text{Now } CV_A = 100 \times \frac{S_1}{\bar{x}_1} = \frac{100 \times 10}{4800} = 0.21$$

$$\text{and } CV_B = 100 \times \frac{S_2}{\bar{x}_2} = \frac{100 \times 12}{5000} = 0.24$$

Thus we conclude that factory A has more consistent structure.

Quartile Deviation: Another measure of dispersion is provided by quartile deviation or semi - inter - quartile range which is given by

$$Q_d = \frac{Q_3 - Q_1}{2}$$

A relative measure of dispersion using quartiles is given by coefficient of quartile deviation which is

$$\text{Coefficient of quartile deviation} = \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$$

Merits

- 1) Quartile deviation provides the best measure of dispersion for open-end classification.
- 2) It is also less affected due to sampling fluctuations.
- 3) Like other measures of dispersion, quartile deviation remains unaffected due to a change of origin but is affected in the same ratio due to change in scale.

Example 26: The quartiles of a variable are 45, 52, and 65 respectively. Its quartile deviation is:

$$\text{Solution: Quartile Deviation} = \frac{Q_3 - Q_1}{2} = \frac{65 - 45}{2} = 10$$

Example 27: If x and y are related as $3x + 4y = 20$ and the quartile deviation of x is 12, then the quartile deviation of y is

Solution: If $y = ax + b$

$$Q.D. \text{ of } y = a \times (Q.D. \text{ of } x)$$

$$3x + 4y = 20$$

$$\text{then } y = \frac{-3}{4}x + 5$$

$$Q.D. \text{ of } y = (3/4) (Q.D. \text{ of } x)$$

$$= |(-3/4)| 12 = 9.$$

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Example 28: Following are the marks of the 10 students : 56, 48, 65, 35, 42, 75, 82, 60, 55, 50. Find quartile deviation and also its coefficient.

Solution: After arranging the marks in an ascending order of magnitude, we get 35, 42, 48, 50, 55, 56, 60, 65, 75, 82

First quartile observation $Q_1 = \frac{(n+1)}{4}$ th observation = $\frac{(10+1)}{4}$ th observation
 = 2.75th observation
 = 2nd observation + 0.75 × difference between the third and the 2nd observation.
 = 42 + 0.75 × (48 – 42)
 = 46.50
 Third quartile (Q_3) = $\frac{3(n+1)}{4}$ th observation
 = 8.25th observation
 = 65 + 0.25 × 10
 = 67.50

Thus applying, we get the quartile deviation as

$$\frac{Q_3 - Q_1}{2} = \frac{67.50 - 46.50}{2} = 10.50$$

Also, using the coefficient of quartile deviation = $\frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$
 = $\frac{67.50 - 46.50}{67.50 + 46.50} = 18.42$

Example 29: If the quartile deviation of x is 6 and $3x + 6y = 20$, what is the quartile deviation of y?

Solution: $3x + 6y = 20$

$$y = \left(\frac{20}{6}\right) + \left(\frac{-3}{6}\right)x$$

Therefore, quartile deviation of y = $\frac{-3}{6} \times$ quartile deviation of X
 = $\frac{1}{2} \times 6 = 3$

Example 30: Find an appropriate measures of dispersion from the following data:

Daily wages (₹)	upto 20	20-40	40-60	60-80	80-100
No. of workers	5	11	14	7	3

Solution: Since this is an open-end classification, the appropriate measure of dispersion would be quartile deviation as quartile deviation does not taken into account the first twenty five percent and the last twenty five per cent of the observations.

Here a denotes the first Class Boundary

Daily wages in ₹ (Class boundary)	No. of workers (less than cumulative frequency)
a	0
20	5
40	16
60	30
80	37
100	40

$$Q_1 = \left[20 + \frac{10 - 5}{16 - 5} \times 20 \right] = ₹ 29.09$$

$$Q_3 = \left[40 + \frac{30 - 16}{30 - 16} \times 20 \right] = 60$$

Thus quartile deviation of wages is given by = $\frac{Q_3 - Q_1}{2} = \frac{₹ 60 - ₹ 29.09}{2} = ₹ 15.46$

Example 31: The mean and variance of 5 observations are 4.80 and 6.16 respectively. If three of the observations are 2, 3 and 6, what are the remaining observations?

Solution: Let the remaining two observations be a and b, then as given

$$\frac{2+3+6+a+b}{5}$$

$$\Rightarrow 11+a+b=24 \Rightarrow a+b=13 \dots\dots\dots (1)$$

$$\text{and } \frac{2^2+a^2+b^2+3^2+6^2}{5} - (4.80)^2 \Rightarrow \frac{49^2+a^2+b^2}{5} - 23.04 = 6.16$$

$$\Rightarrow 49+a^2+b^2=146$$

$$\Rightarrow a^2+b^2=97 \dots\dots\dots (2)$$

$$\text{From (1), we get } a = 13 - b \dots\dots\dots (3)$$

Eliminating a from (2) and (3), we get

$$(13 - b)^2 + b^2 = 97 \Rightarrow 169 - 26b + 2b^2 = 97$$

$$\Rightarrow b^2 - 13b + 36 = 0$$

$$\Rightarrow (b-4)(b-9) = 0$$

$$\Rightarrow b = 4 \text{ or } 9$$

$$\text{From (3), } a = 9 \text{ or } 4$$

Thus the remaining observations are 4 and 9.

Example 32: If Standard deviation of x is σ , then standard deviation of $\frac{ax+b}{c}$, where a, b and c are constants, will be, then SD of y will be

Solution: SD of X = σ .

$$\text{Let } y = \frac{ax+b}{c} = \frac{ax}{c} + \frac{b}{c}$$

$$y = \frac{b}{c} + \frac{ax}{c}$$

$$\text{SD of } y = \left| \frac{a}{c} \right| \text{SD of } x = \left| \frac{a}{c} \right| \cdot \sigma$$

Example 33: Find at the variance given arithmetic mean = $\frac{(8+4)}{2}$

Solution: Here Largest Value (L) = 8

Smallest Value (S) = 4

$$\text{Range} = \text{Largest Value} - \text{Smallest Value} = 8 - 4 = 4$$

$$\text{We know that SD} = \frac{\text{Range}}{2} = \frac{4}{2} = 2$$

$$\text{Variance} = (\text{SD})^2 = (2)^2 = 4$$

Example 34: If Mean and coefficient of variation of the marks of 10 students is 20 and 80, respectively. What will be the variance of them?

Solution: Given No. of observations (N) = 10

$$\text{Mean } (\bar{x}) = 20$$

$$\text{CV} = 80$$

$$\text{CV} = \frac{\text{SD}}{\text{AM}} \times 100$$

$$80 = \frac{\text{SD}}{20} \times 100$$

$$\text{SD} = \frac{80 \times 20}{100} = 16$$

$$\text{Variance} = (\text{SD})^2 = (16)^2 = 256$$

Example 35: If arithmetic mean and coefficient of variation x are 10 and 40 respectively then variance of $15 - \frac{3x}{2}$ will be

Solution: Given Mean of x = 10, Coefficient of Variation of (x) = 40

$$\text{C V of X} = \frac{\text{SD of X}}{\text{Mean of X}} \times 100$$

$$40 = \text{SD of } x \times 100$$

$$\text{SD of } x = \frac{400}{100} = 4$$

$$\text{Now } y = 15 - \frac{3x}{2}$$

$$2y = -30 + 3x \quad \therefore 2y = 3x - 30$$

$$\therefore y = \frac{3x}{2} - \frac{30}{2} \quad \therefore y = \frac{3x}{2} - 15$$

$$\therefore 3x - 2y - 30 = 0$$

$$\text{S.D of } y = |b| \text{S.D of } X$$

$$\text{S.D of } y = \left| \frac{3}{2} \right| \times 4 = 6$$

$$\text{Variance of } y = (6)^2 = 36$$

Example 36: Coefficient of Quartile deviation is $\frac{1}{4}$ then $\frac{Q_3}{Q_1}$ is

Solution: Coefficient of QD = $\frac{1}{4}$

$$\frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{1}{4}$$

$$4Q_3 - 4Q_1 = Q_3 + Q_1$$

$$\frac{Q_3}{Q_1} = \frac{5}{3}$$

Example 37: SD from numbers 1, 4, 5, 7, 8 is 2.45. If 10 is added to each then SD will be

Solution: We know a change in origin of SD have no change in SD. So, New SD = Original Sd when 10 will be added, So, SD will not change.

Example 38: A student computes the AM and SD for a set of 100 observations as 50 and 5 respectively. Later on, she discovers that she has made a mistake in taking one observation as 60 instead of 50. What would be the correct mean and SD if

- i) The wrong observation is left out?
- ii) The wrong observation is replaced by the correct observation?

Solution: As given, $n = 100$, $\bar{x} = 50$, $S = 5$

Wrong observation = 60(x), correct observation = 50(V)

$$\bar{x} = \frac{\sum x_i}{n}$$

$$\therefore \sum x_i = n\bar{x} = 100 \times 50 = 5000$$

$$\text{and } s^2 = \frac{\sum x_i^2}{n} - \bar{x}^2$$

$$\therefore \sum x_i^2 = n(\bar{x}^2 + s^2) = 100(50^2 + 5^2) = 252500$$

- i) Sum of the 99 observations = $5000 - 60 = 4940$
 AM after leaving the wrong observation = $4940/99 = 49.90$
 Sum of squares of the observation after leaving the wrong observation
 = $252500 - 60^2 = 248900$
 Variance of the 99 observations = $248900/99 - (49.90)^2$
 = $2514.14 - 2490.01$
 = 24.13
 \therefore SD of 99 observations = 4.91
- ii) Sum of the 100 observations after replacing the wrong observation by the correct observation = $5000 - 60 + 50 = 4990$
 AM = $\frac{4990}{100} = 49.90$
 Corrected sum of squares = $252500 + 50^2 - 60^2 = 251400$

$$\begin{aligned} \text{Corrected SD} &= \sqrt{\frac{251400}{100} - (49.90)^2} \\ &= \sqrt{23.94} = 4.90 \end{aligned}$$

Example 39: Compute coefficient of variation from the following data:

Age	under 10	under 20	under 30	under 40	under 50	under 60
No. of persons Dying	10	18	30	45	60	80

Solution: Given in this problem is less than cumulative frequency distribution. We need first convert it to a frequency distribution and then compute the coefficient of variation.

Table : Computation of coefficient of variation

Class Interval Age in years	No. of persons dying	Mid-value	$d_i = \frac{x_i - 25}{10}$	$f_i d_i$	$f_i d_i^2$
	(f_i)	(x_i)	10		
0-10	10	5	-2	-20	40
10-20	18-10=8	15	-1	-8	8
20-30	30-18=12	25	0	0	0
30-40	45-30=15	35	1	15	15
40-50	60-45=15	45	2	30	60
50-60	80-60=20	55	3	60	180
Total	80	-	-	77	303

The AM is given by:

$$\begin{aligned} \bar{x} &= A + \frac{\sum f_i d_i}{N} \times C \\ &= \left(25 + \frac{77}{80} \times 10 \right) \text{ years} \\ &= 34.63 \text{ years} \end{aligned}$$

The standard deviation is

$$\begin{aligned} s &= \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N} \right)^2} \times C \\ &= \sqrt{\frac{303}{80} - \left(\frac{77}{80} \right)^2} \times 10 \text{ years} \\ &= \sqrt{3.79 - 0.93} \times 10 \text{ years} \\ &= 16.91 \text{ years} \end{aligned}$$

Thus the coefficient of variation is given by

$$\begin{aligned} \text{CV} &= \frac{S}{\bar{x}} \times 100 \\ &= \frac{16.91}{34.63} \times 100 = 48.83 \end{aligned}$$

Comparison between different measures of dispersion

We may now have a review of the different measures of dispersion on the basis of their relative merits and demerits.

1. Standard deviation, like AM, is the best measure of dispersion. It is rigidly defined, based on all the observations, not too difficult to compute, not much affected by sampling fluctuations and moreover it has some desirable mathematical properties. All these merits of standard deviation make SD as the most widely and commonly used measure of dispersion.
2. Range is the quickest to compute and as such, has its application in statistical quality control. However, range is based on only two observations and affected too much by the presence of extreme observation(s).
3. Mean deviation is rigidly defined, based on all the observations, and not much affected by sampling fluctuations. However, mean deviation is difficult to comprehend and its computation is also time consuming and laborious. Furthermore, unlike SD, mean deviation does not possess mathematical properties.
4. Quartile deviation is also rigidly defined, easy to compute and not much affected by sampling fluctuations. The presence of extreme observations has no impact on quartile deviation since quartile deviation is based on the central fifty-percent of the observations. However, quartile deviation is not based on all the observations and it has no desirable mathematical properties. Nevertheless, quartile deviation is the best measure of dispersion for open-end classifications.