# Marathon 5

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NISH10

# Schedule

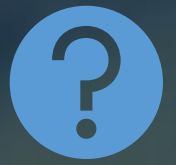
| Date (Day) | Topic |
|---|---|
| 12-06-2023 (Monday) | Time Value of Money |
| 13-06-2023 (Tuesday) | Logical Reasoning |
| 14-06-2023 (Wednesday) | Measures of Central Tendency and Dispersion |
| 15-06-2023 (Thursday) | Ratio, Proportion, Indices, Logarithms; Linear Inequalities |
| 16-06-2023 (Friday) | Equations; Statistical Description of Data |
| 17-06-2023 (Saturday) | Sequence and Series |
| 18-06-2023 (Sunday) | Sets, Relations, and Functions |
| 19-06-2023 (Monday) | Correlation and Regression |
| 20-06-2023 (Tuesday) | Index Numbers |
| 21-06-2023 (Wednesday) | Permutations and Combinations |
| 22-06-2023 (Thursday) | Probability |
| 23-06-2023 (Friday) | Theoretical Distributions |

# Highlights

Conceptual Revision

Question Based Revision

Last Day Preparation Tips



Questions to Revise on the day before Exam

# Chapter 2 – Equations

# Concepts at a Glance

## Simple Equations

- An equation with only one variable with a linear power is known as a Simple Equation.
- Try the options to solve the questions.

## Simultaneous Linear Equations in Two Variables

- Two equations with two variables with linear power are known as simultaneous linear equations.
- Try the options to solve the questions.

## Quadratic Equations

- A quadratic equation is an equation in which the highest power of the variables is 2.
- A quadratic equation is of the form $ax^2 + bx + c = 0$.
- $x$ is a variable while $a$, $b$ and $c$ are constants.
- A quadratic equation has two solutions/roots.

## Methods of Solving Quadratic Equations

There are three methods of solving any quadratic equation:

1. Factorization Method
2. Quadratic Formula

Quadratic Formula = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

If we call the roots $\alpha$, and $\beta$, then,

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Sum of Roots $(\alpha + \beta) = -\frac{b}{a}$

Product of Roots $\alpha\beta = \frac{c}{a}$

3. Fastest Method

Solve the equation $x^2 - 5x + 6 = 0$ using fastest method.

Here, $a = 1$; $b = -5$; $c = 6$

Sum of Roots $= -\frac{b}{a} = -\frac{-5}{1} = 5$

Product of Roots $= \frac{c}{a} = \frac{6}{1} = 6$

Now, take the sum of the roots, divide it by half, and add $x$ to it. You'll get $\left(\frac{5}{2} + x\right)$.

Similarly, take the sum of the roots, divide it by half, and subtract $x$ from it. You'll get $\left(\frac{5}{2} - x\right)$. Multiply these two and equate with the product, i.e. 6.

$$\left(\frac{5}{2}+x\right)\left(\frac{5}{2}-x\right)=6$$

$$\Rightarrow \left(\frac{5}{2}\right)^2 - x^2 = 6$$

$$\Rightarrow \frac{25}{4} - x^2 = 6$$

$$\Rightarrow x^2 = \frac{25}{4} - 6$$

$$\Rightarrow x^2 = 6.25 - 6$$

$$\Rightarrow x^2 = 0.25$$

$$\Rightarrow x = \sqrt{0.25}$$

$$\Rightarrow x = 0.5$$

Now, put the value of $x = 0.5$ in the factors $\left(\frac{5}{2}+x\right)$, and $\left(\frac{5}{2}-x\right)$. You'll get the roots.

Therefore, $\alpha = \frac{5}{2}+0.5 = 3;\ \beta = \frac{5}{2}-0.5 = 2.$

This method applies to complicated roots as well.

## Important Rule

If $\alpha$ and $\beta$ are the roots of the equation, the equation is given by:

$$x^2 - (\alpha+\beta)x + \alpha\beta = 0$$

In other words,

$$x^2 - (Sum\,of\ Roots)\,x + Product\,of\ Roots = 0.$$

## Nature of Roots

We know that the quadratic formula gives us the value of $x$ as follows:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

In this formula, the term $b^2 - 4ac$ plays a very important role. The nature of the roots is dependent on $b^2 - 4ac$.

1. If $b^2 - 4ac = 0$, the roots are real and equal.
2. If $b^2 - 4ac > 0$, the roots are real and unequal.
    a. If $b^2 - 4ac$ is a perfect square, the roots are real, rational, and unequal.

b. If $b^2 - 4ac$ is not a perfect square, the roots are real, irrational, and unequal.

3. If $b^2 - 4ac < 0$, the roots are imaginary and unequal.

Since $b^2 - 4ac$ discriminates the roots, it is known as the discriminant.

**Points to be noted –**

1. A real number is a number which can be expressed on a number line. Therefore, every number is a real number, including negative numbers.
2. An imaginary number is a number multiplied by a unit "*i*", which is identified by its property $i^2 = -1$.
3. An integer is a number without any fractional part. It includes positive as well as negative numbers.

4. A rational number is a number which can be expressed as a fraction of two integers. The decimal expansion of a rational number either terminates after a finite number of digits, or begins to repeat the same finite sequence of digits over and over. Examples:

   a. 2 is a rational number as it can be expressed in the form of $\frac{2}{1}$.

   b. $\frac{5}{2}$ is a rational number as its decimal expansion 2.5 terminates after a finite number of digits.

   c. $\frac{2}{9}$ is a rational number as its decimal expansion comes to 0.222…, i.e. it begins to repeat itself over and over.

d. $-\frac{5}{2}, -\frac{2}{9}$ are also rational numbers.

5. An irrational number is a number whose decimal expansion either does not terminate after a finite number of digits or does not repeat itself over and over.
   Examples:
   a. $\pi$ is an irrational number as its decimal expansion is 3.14159265359…, i.e. it neither terminates after a finite number of digits nor does it repeat itself over and over.
   b. $\sqrt{2}$ is an irrational number as its decimal expansion is 1.41421356237…, i.e. it neither terminates after a finite number of digits nor does it repeat itself over and over.

6. Irrational roots occur in conjugate pairs, i.e. if $\left(m+\sqrt{n}\right)$ is a root, then $\left(m-\sqrt{n}\right)$ is the other root of the same equation.
7. If one root is reciprocal to the other root, then their product is 1 and so $\frac{c}{a}=1$, i.e. $c=a$.
8. If one root is equal to the other root but opposite in sign, then their sum = 0, i.e. $-\frac{b}{a}=0\Rightarrow b=0$.

## Cubic Equations

- An equation with the highest power of the variables as 3 is known as a cubic equation.

- Try the options to solve such an equation.

# Questions Based on Simple Equations

**Question 1**

The denominator of a fraction exceeds the numerator by 5 and if 3 be added to both the fraction becomes $\frac{3}{4}$. Find the fraction.

(a) $\frac{12}{17}$ (b) $\frac{13}{17}$ (c) $\frac{14}{18}$ (d) $\frac{15}{19}$

**Question 2**

If thrice of A's age 6 years ago, be subtracted from twice his present age, the result would be equal to his present age. Find A's present age.

(a) 8 (b) 9 (c) 10 (d) 11

**Question 3**

A number consists of two digits. The digit in the ten's place is twice the digit in the unit's place. If 18 be subtracted from the number, the digits are reversed. Find the number.

(a) 63 (b) 84 (c) 42 (d) 21

**Question 4**

For a certain commodity, the demand equation giving demand *'d'* in kg, for a price *'p'* in rupees per kg. is $d = 100(10 - p)$. The supply equation giving the supply $s$ in kg. for a price $p$ in rupees per kg. is $s = 75(p - 3)$. The market price is such at which demand equals supply. Find the market price and quantity that will be bought and sold.

(a) 10, 400, 400 (b) 9, 500, 500 (c) 8, 340, 440 (d) 7, 300, 300

**Question 5**

The sum of two numbers is 52 and their difference is 2. The numbers are:

(a) 17 and 15 (b) 12 and 10 (c) 27 and 25 (d) None

**Question 6**

The diagonal of a rectangle is 5 cm and one of at sides is 4 cm. Its area is:

(a) 20 sq. cm. (b) 12 sq. cm. (c) 10 sq. cm. (d) None

**Question 7**

Divide 56 into two parts such that three times the first part exceeds one third of the second by 48. The parts are:

(a) (20, 36) (b) (25, 31) (c) (24, 32) (d) None

**Question 8**

The sum of the digits of a two-digit number is 10. If 18 be subtracted from it, the digits in the resulting number will be equal. The number is:

(a) 37 (b) 73 (c) 75 (d) None

**Question 9**

The fourth part of a number exceeds the sixth part by 4. The number is:

(a) 84 (b) 44 (c) 48 (d) None

**Question 10**

Ten years ago, the age of a father was four times of his son. Ten years hence, the age of the father will be twice that of his son. The present ages of the father and the son are:

(a) (50, 20) (b) (60, 20) (c) (55, 25) (d) None

**Question 11**

The product of two numbers is 3200 and the quotient when the larger number is divided by the smaller is 2. The numbers are:

(a) (16, 200) (b) (160, 20) (c) (60, 30) (d) (80, 40)

**Question 12**

The denominator of a fraction exceeds the numerator by 2. If 5 be added to the numerator, the fraction increases by unity. The fraction is:

(a) $\frac{5}{7}$ (b) $\frac{1}{3}$ (c) $\frac{7}{9}$ (d) $\frac{3}{5}$

**Question 13**

Three persons Mr. Roy, Mr. Paul and Mr. Singh together have ₹51. Mr. Paul has ₹4 less than Mr. Roy and Mr. Singh has got ₹5 less than Mr. Roy. They have the money as:

(a) (₹20, ₹16, ₹15) (b) (₹15, ₹20, ₹16) (c) (₹25, ₹11, ₹15) (d) None

**Question 14**

A number consists of two digits. The digits in the ten's place is 3 times the digit in the unit's place. If 54 is subtracted from the number, the digits are reversed. The number is:

(a) 39 (b) 92 (c) 93 (d) 94

**Question 15**

One student is asked to divide a half of a number by 6 and other half by 4 and then to add the two quantities. Instead of doing so, the student divides the given number by 5. If the answer is 4 short of the correct answer, then the number was:

(a) 320 (b) 400 (c) 480 (d) None

**Question 16**

If a number of which the half is greater than $1/5^{th}$ of the number by 15, then the number is:

(a) 50 (b) 40 (c) 80 (d) None

# Questions Based on Simultaneous Linear Equations in Two Variables

**Question 16**

The point of intersection between the lines $3x+4y=7$ and $4x-y=3$ lie in the:

(a) 1st Quadrant (b) 2nd Quadrant (c) 3rd Quadrant (d) 4th Quadrant

**Question 17**

If the numerator of a fraction is increased by 2 and the denominator by 1, it becomes 1. Again, if the numerator is decreased by 4 and the denominator by 2, it becomes 1/2. Find the fraction.

(a) 2/3 (b) 4/5 (c) 7/8 (d) None

**Question 18**

The age of a man is three times the sum of the ages of his two sons and 5 years hence his age will be double the sum of their ages. Find the present age of the man?

(a) 23 (b) 45 (c) 78 (d) None

**Question 19**

A number consist of three digits of which the middle one is zero and the sum of the other digits is 9. The number formed by interchanging the first and third digits is more than the original number by 297. Find the number.

(a) 306 (b) 207 (c) 702 (d) None

**Question 20**

Monthly incomes of two persons are in the ratio 4 : 5 and their monthly expenses are in the ratio 7 : 9. If each saves ₹50 per month find their monthly incomes.

(a) (500, 400) (b) (400, 500) (c) (300, 600) (d) (350, 550)

**Question 21**

Find the fraction which is equal to 1/2 when both its numerator and denominator are increased by 2. It is equal to 3/4 when both are increased by 12.

(a) 3/8 (b) 5/8 (c) 2/8 (d) 2/3

**Question 22**

The age of a person is twice the sum of the ages of his two sons and five years ago his age was thrice the sum of their ages. Find his present age.

(a) 60 years (b) 52 years (c) 51 years (d) 50 years

**Question 23**

A number between 10 and 100 is five times the sum of its digits. If 9 be added to it the digits are reversed find the number.

(a) 54 (b) 53 (c) 45 (d) 55

**Question 24**

The wages of 8 men and 6 boys amount to ₹33. If 4 men earn ₹4.50 more than 5 boys determine the wages of each man and boy.

(a) (₹1.50, ₹3) (b) (₹3, ₹1.50) (c) (₹2.50, ₹2) (d) (₹2, ₹2.50)

**Question 25**

A number consisting of two digits is four times the sum of its digits and if 27 be added to it the digits are reversed. The number is:

(a) 63 (b) 35 (c) 36 (d) 60

**Question 26**

Of two numbers, 1/5th of the greater is equal to 1/3rd of the smaller and their sum is 16. The numbers are:

(a) (6, 10) (b) (9, 7) (c) (12, 4) (d) (11, 5)

**Question 27**

$y$ is older than $x$ by 7 years. 15 years back, $x$'s age was $3/4^{th}$ of $y$'s age. Their present ages are:

(a) ($x = 36, y = 43$) (b) ($x = 50, y = 43$) (c) ($x = 43, y = 50$) (d) ($x = 40, y = 47$)

**Question 28**

The sum of the digits in a three digit number is 12. If the digits are reversed, the number is increased by 495 but reversing only of the tens and units digits increases the number by 36. The number is:

(a) 327 (b) 372 (c) 237 (d) 273

**Question 29**

Two numbers are such that twice the greater number exceeds twice the smaller one by 18 and 1/3$^{rd}$ of the smaller and 1/5$^{th}$ of the greater number are together 21. The numbers are:

(a) (36, 45) (b) (45, 36) (c) (50, 41) (d) (55, 46)

**Question 30**

The demand and supply equations for a certain commodity are $4q + 7p = 17$ and $p = \frac{q}{3} + \frac{7}{4}$ respectively where $p$ is the market price and $q$ is the quantity. The equilibrium price and quantity are:

(a) $2, \frac{3}{4}$ (b) $3, \frac{1}{2}$ (c) $5, \frac{3}{5}$ (d) None

**Question 31**

The cab bill is partly fixed and partly varies on the distance covered. For 456 km, the bill is ₹8252, for 484 km the bill is ₹8728. What will the bill be for 500 km?

(a) ₹8876 (b) ₹9156 (c) ₹9472 (d) ₹9000

## Question 31

**Question 32**

The value of $k$ for the system of equations $kx+2y=5$ and $3x+y=1$ has no solution is:

(a) 5 (b) 2/3 (c) 6 (d) 3/2

## Question 32

# Questions Based on Quadratic Equations

**Question 33**

If $\alpha$, $\beta$ be the roots of the equation $2x^2 - 4x - 3 = 0$, then the value of $\alpha^2 + \beta^2$ is:

(a) 5 (b) 7 (c) 3 (d) –4

## Question 33

**Question 34**

Examine the nature of roots of the following equation: $3x^2 - 8x + 4 = 0$.

(a) Real and Equal
(b) Real and Unequal
(c) Imaginary and Unequal
(d) Real, Rational, Unequal

**Question 35**

Examine the nature of roots of the following equation: $5x^2 - 4x + 2 = 0$.

(a) Real and Equal
(b) Real and Unequal
(c) Imaginary and Unequal
(d) Real, Rational, Unequal

**Question 36**

If the roots of the equation $2x^2 + 8x - m^3 = 0$ are equal, then the value of $m$ is:

(a) –3 (b) –1 (c) 1 (d) –2

**Question 37**

The equation $x^2-(p+4)x+2p+5=0$ has equal roots. The value of $p$ will be:

(a) $\pm 1$ (b) 2 (c) $\pm 2$ (d) –2

## Question 37

**Question 38**

The roots of the equation $x^2+(2p-1)x+p^2=0$ are real if:

(a) $p \geq 1$ (b) $p \leq 4$ (c) $p \geq 1/4$ (d) $p \leq 1/4$

**Question 39**

If $L+M+N=0$, and L, M, and N are rationals, the roots of the equation $(M+N-L)x^2+(N+L-M)x+(L+M-N)=0$ are:

(a) Real and Irrational
(b) Real and Rational
(c) Imaginary and Equal
(d) Real and Equal

**Solution**

(b)

We have

$$(M+N-L)x^2+(N+L-M)x+(L+M-N)=0$$

We know that

$$L+M+N=0$$

Therefore,

$$M+N=-L;\ N+L=-M;\ L+M=-N;\ M=-N-L$$

Therefore, we have

$$(-L-L)x^2+(-M-M)x+(-N-N)=0$$

$$\Rightarrow -2Lx^2-2Mx-2N=0$$

$$\Rightarrow -2\left(Lx^2+Mx+N\right)=0$$

$$\Rightarrow Lx^2+Mx+N=0$$

Here, $a=L;\ b=M;\ c=N$

$$b^2-4ac=M^2-(4)(L)(N)$$

$$=(-N-L)^2-4LN$$

$$=\{-(N+L)\}^2-4LN$$

$$= (N + L)^2 - 4LN$$

$$= N^2 + L^2 + 2LN - 4LN$$

$$= N^2 + L^2 - 2LN$$

$$= (N - L)^2$$

Therefore, *D* is a perfect square. Hence, the roots are rational. Also, the roots are real. This is because even if N – L comes to be a negative figure, squaring it would make it positive, and thereafter, its square root will be determined in the quadratic formula. Therefore, the roots are Real and Rational.

**Question 40**

If one root of the equation is $2-\sqrt{3}$, form the equation given that the roots are irrational.

(a) $x^2-4x+2=0$

(b) $x^2-3x+9=0$

(c) $x^2-5x+2=0$

(d) $x^2-4x+1=0$

**Question 41**

If the roots of the equation $p(q-r)x^2+q(r-p)x+r(p-q)=0$ are equal, find the value of $\frac{1}{p}+\frac{1}{r}$.

(a) $\frac{2}{q}$ (b) $\frac{1}{q}$ (c) $\frac{1}{2}$ (d) None

**Solution**

(a)

Here, $a = p(q-r);\ b = q(r-p);\ c = r(p-q)$

Since the roots of this equation are equal, $b^2 - 4ac = 0$.

$$\{q(r-p)\}^2 - (4)\{p(q-r)\}\{r(p-q)\} = 0$$

$$q^2(r-p)^2 - [4pr(q-r)(p-q)] = 0$$

$$q^2(r^2 + p^2 - 2rp) - [4pr(qp - q^2 - pr + qr)] = 0$$

$$q^2r^2 + q^2p^2 - 2rpq^2 - [4p^2qr - 4pq^2r - 4p^2r^2 + 4pqr^2] = 0$$

$$q^2r^2+q^2p^2-2rpq^2-4p^2qr+4pq^2r+4p^2r^2-4pqr^2=0$$

$$q^2r^2+q^2p^2+4pq^2r-2rpq^2-4p^2qr+4p^2r^2-4pqr^2=0$$

$$q^2r^2+q^2p^2+2pq^2r-4p^2qr+4p^2r^2-4pqr^2=0$$

We know that $(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$

If we look closely at the LHS of the following equation, $q^2r^2+q^2p^2+2pq^2r-4p^2qr+4p^2r^2-4pqr^2=0$, we'll find that it is the expansion of $(qr+qp-2pr)^2$.

Therefore,

$$(qr+qp-2pr)^2=0$$

$$\Rightarrow qr + qp - 2pr = 0$$

$$\Rightarrow qr + qp = 2pr$$

Dividing the entire equation by $pqr$, we get:

$$\frac{qr}{pqr} + \frac{qp}{pqr} = \frac{2pr}{pqr}$$

$$\Rightarrow \frac{1}{p} + \frac{1}{r} = \frac{2}{q}$$

**Question 42**

If $\alpha$ and $\beta$ be the roots of $x^2 + 7x + 12 = 0$, find the equation whose roots are $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$.

(a) $x^2+50x+49=0$

(b) $x^2-24x+144=0$

(c) $x^2-50x+49=0$

(d) $x^2-19x+49=0$

## Question 42

**Question 43**

If $\alpha$, $\beta$ are the two roots of the equation $x^2 + px + q = 0$, form the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

(a) $qx^2 - \left(p^2 - 2q\right)x + q = 0$

(b) $px^2 - \left(p^2 - 2q\right)x + q = 0$

(c) $qx^2 - \left(p^2 - 2q\right)x + p = 0$

(d) $qx^2 + \left(p^2 - 2q\right)x + p = 0$

**Solution**

(a)

$x^2 + px + q = 0$

$\Rightarrow \alpha + \beta = -\frac{b}{a} = -\frac{p}{1} = -p$, and

$\alpha\beta = \frac{c}{a} = \frac{q}{1} = q$

We need an equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$

Quadratic equation is given by: $x^2 - (Sum\,of\ Roots)x + Product\,of\ Roots = 0$

Therefore,

$$x^2-\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)x+\left(\frac{\alpha}{\beta}\times\frac{\beta}{\alpha}\right)=0$$

$$\Rightarrow x^2-\left(\frac{\alpha^2+\beta^2}{\alpha\beta}\right)x+1=0$$

$$\Rightarrow x^2-\left(\frac{\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta}{\alpha\beta}\right)x+1=0$$

$$\Rightarrow x^2-\left\{\frac{\left(\alpha^2+\beta^2+2\alpha\beta\right)-2\alpha\beta}{\alpha\beta}\right\}x+1=0$$

$$\Rightarrow x^2 - \left\{\frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}\right\}x + 1 = 0$$

$$\Rightarrow x^2 - \left\{\frac{(-p)^2 - (2q)}{q}\right\}x + 1 = 0$$

$$\Rightarrow x^2 - \left\{\frac{p^2 - 2q}{q}\right\}x + 1 = 0$$

Multiplying the entire equation with $q$, we get:

$$qx^2 - q\left\{\frac{p^2 - 2q}{q}\right\}x + q = 0$$

$$\Rightarrow qx^2-\left(p^2-2q\right)x+q=0$$

**Question 44**

If one root of $5x^2 + 13x + p = 0$ be reciprocal of the other, then the value of $p$ is:

(a) –5 (b) 5 (c) 1/5 (d) –1/5

**Question 45**

If $\alpha$ and $\beta$ are the roots of $x^2 = x + 1$, then the value of $\frac{\alpha^2}{\beta} - \frac{\beta^2}{\alpha}$ is:

(a) $2\sqrt{5}$
(b) $\sqrt{5}$
(c) $3\sqrt{5}$
(d) $-2\sqrt{5}$

## Question 45

**Question 46**

If $\alpha, \beta$ be the roots of $2x^2 - 4x - 1 = 0$, find the value of $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$.

(a) –22 (b) 23 (c) –23 (d) None

**Question 47**

The value of $4+\cfrac{1}{4+\cfrac{1}{4+\cfrac{1}{4+\ldots\infty}}}$ is:

(a) $1\pm\sqrt{2}$ (b) $2+\sqrt{5}$ (c) $2\pm\sqrt{5}$ (d) None

## Question 47

**Question 48**

The value of $\sqrt{6+\sqrt{6+\sqrt{6+...\infty}}}$ is:

(a) –3 (b) 2 (c) 3 (d) 4

## Question 48

**Question 49**

If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then $\frac{b^2}{ac} + \frac{bc}{a^2}$ is:

(a) 2 (b) –2 (c) 1 (d) –1

**Solution**

(a)

$$ax^2 + bx + c = 0$$

$$\alpha + \beta = -\frac{b}{a}$$

$$\alpha\beta = \frac{c}{a}$$

Given: $\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$$-\frac{b}{a}=\frac{\beta^2+\alpha^2}{\alpha^2\beta^2}$$

$$-\frac{b}{a}=\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$$

$$-\frac{b}{a}=\frac{\left(-\frac{b}{a}\right)^2-\left(2\times\frac{c}{a}\right)}{\left(\frac{c}{a}\right)^2}$$

$$-\frac{b}{a}=\left(\frac{b^2}{a^2}-\frac{2c}{a}\right)\div\frac{c^2}{a^2}$$

$$-\frac{b}{a}=\left(\frac{b^2-2ac}{a^2}\right)\times\frac{a^2}{c^2}$$

$$-\frac{b}{a}=\frac{b^2-2ac}{c^2}$$

$$-bc^2=ab^2-2a^2c$$

$$ab^2+bc^2=2a^2c$$

Dividing both sides by $a^2c$

$$\frac{ab^2}{a^2c}+\frac{bc^2}{a^2c}=\frac{2a^2c}{a^2c}$$

$$\frac{b^2}{ac}+\frac{bc}{a^2}=2$$

**Question 50**

If $p$ and $q$ are the roots of the $x^2+2x+1=0$, then the values of $p^3+q^3$ becomes:

(a) 2 (b) –2 (c) 4 (d) –4

**Question 51**

If $p \neq q$ and $p^2 = 5p - 3$ and $q^2 = 5q - 3$, the equation having the roots as $\frac{p}{q}$ and $\frac{q}{p}$ is:

(a) $x^2 - 19x + 3 = 0$

(b) $3x^2 - 19x - 3 = 0$

(c) $3x^2 - 19x + 3 = 0$

(d) $3x^2 + 19x + 3 = 0$

**Solution**

(c)

We have $p^2 = 5p - 3$

$p^2 - 5p + 3 = 0$

$a = 1;\ b = -5;\ c = 3$

$$\alpha + \beta = -\frac{b}{a} = -\frac{-5}{a} = 5$$

$$\alpha\beta = \frac{c}{a} = \frac{3}{1} = 3$$

$$\left(\frac{5}{2}+x\right)\left(\frac{5}{2}-x\right)=3$$

$$\left(\frac{5}{2}\right)^2-x^2=3$$

$$x^2=\frac{25}{4}-3$$

$$x^2=\frac{25-12}{4}=\frac{13}{4}$$

$$x=\sqrt{\frac{13}{4}}=\frac{\sqrt{13}}{2}$$

$$\alpha = \frac{5}{2} + \frac{\sqrt{13}}{2} = \frac{5+\sqrt{13}}{2}$$

$$\beta = \frac{5}{2} - \frac{\sqrt{13}}{2} = \frac{5-\sqrt{13}}{2}$$

Therefore, $p = \frac{5+\sqrt{13}}{2}, \frac{5-\sqrt{13}}{2}$

Also, we have $q^2 = 5q - 3$

Since this is exactly the same as $p^2 = 5p - 3$, it's obvious that $q$ will also have the same two values.

Since it is given in the question that $p \neq q$, therefore, we'll have to take different values.

So, let $p = \frac{5+\sqrt{13}}{2}$, and $q = \frac{5-\sqrt{13}}{2}$

Now, we need to find the equation whose roots are $\frac{p}{q}$ and $\frac{q}{p}$.

$$\frac{p}{q} = \frac{\frac{5+\sqrt{13}}{2}}{\frac{5-\sqrt{13}}{2}} = \frac{5+\sqrt{13}}{5-\sqrt{13}} = 6.1713$$

$$\frac{q}{p}=\frac{\frac{5-\sqrt{13}}{2}}{\frac{5+\sqrt{13}}{2}}=\frac{5-\sqrt{13}}{5+\sqrt{13}}=0.1620$$

If the roots are given, the equation is given by:

$$x^2-\left(Sum\,of\ Roots\right)x+Product\,of\ Roots=0$$

Therefore, the equation is:

$$x^2-\left(6.1713+0.1620\right)x+\left(6.1713\times0.1620\right)=0$$

$$x^2-6.333x+1=0$$

Now, try the options.

Option (a) cannot be the answer.

Option (b) cannot be the answer as the last term has a negative sign.

Option (c) $\rightarrow 3x^2 - 19x + 3 = 0$

Dividing the entire equation by 3, we'll get:

$$\frac{3x^2}{3} - \frac{19}{3}x + \frac{3}{3} = 0$$

$$\Rightarrow x^2 - 6.333x + 1 = 0$$

**Question 52**

If the root of the equation $x^2 - 8x + m = 0$ exceeds the other by 4, then the value of $m$ is:

(a) 10
(b) 11
(c) 9
(d) 12

**Question 53**

If arithmetic mean between roots of a quadratic equation is 8 and the geometric mean between them is 5, the equation is _____.

(a) $x^2 - 16x - 25 = 0$ (b) $x^2 - 16x + 25 = 0$ (c) $x^2 + 16x + 25 = 0$ (d) None

**Question 54**

The harmonic mean of the roots of the equation $\left(5+\sqrt{2}\right)x^2-\left(4+\sqrt{5}\right)x+8+2\sqrt{5}=0$ is:

(a) 2 (b) 4 (c) 6 (d) 8

**Question 55**

Difference between a number and its positive square root is 12; find the numbers.

(a) 4, 16 (b) 16, 4 (c) 22, 35 (d) Both (a) and (b)

**Question 56**

A piece of iron rod costs ₹60. If the rod was 2 metre shorter and each metre costs ₹1.00 more, the cost would remain unchanged. What is the length of the rod?

(a) 10 m (b) 14 m (c) 12 m (d) None

**Question 57**

Divide 25 into two parts so that sum of their reciprocals is 1/6.

(a) 8 and 17 (b) 10 and 15 (c) 20 and 5 (d) None

**Question 58**

The sum of two numbers is 8 and the sum of their squares is 34. Taking one number as $x$ form an equation in $x$ and hence find the numbers. The numbers are:

(a) (7, 10) (b) (4, 4) (c) (3, 5) (d) (2, 6)

**Question 59**

The difference of two positive integers is 3 and the sum of their squares is 89. Taking the smaller integer as $x$ form a quadratic equation and solve it to find the integers. The integers are:

(a) (7, 4) (b) (5, 8) (c) (3, 6) (d) (2, 5)

**Question 60**

Five times of a positive whole number is 3 less than twice the square of the number. The number is

(a) 3 (b) 4 (c) –3 (d) 2

**Question 61**

The area of a rectangular field is 2000 sq.m. and its perimeter is 180 m. Form a quadratic equation by taking the length of the field as $x$ and solve it to find the length and breadth of the field. The length and breadth are:

(a) (205 m, 80 m) (b) (50 m, 40 m) (c) (60 m, 50 m) (d) None

**Question 62**

Two squares have sides $p$ cm and $(p + 5)$ cms. The sum of their squares is 625 sq. cm. The sides of the squares are:

(a) (10 cm, 30 cm) (b) (12 cm, 25 cm) (c) (15 cm, 20 cm) (d) None

**Question 63**

Divide 50 into two parts such that the sum of their reciprocals is 1/12. The numbers are:

(a) (24, 26) (b) (28, 22) (c) (27, 23) (d) (20, 30)

**Question 64**

There are two consecutive numbers such that the difference of their reciprocals is 1/240. The numbers are:

(a) (15, 16) (b) (17, 18) (c) (13, 14) (d) (12, 13)

**Question 65**

The hypotenuse of a right–angled triangle is 20 cm. The difference between its other two sides be 4 cm. The sides are:

(a) (11 cm, 15 cm) (b) (12 cm, 16 cm) (c) (20 cm, 24 cm) (d) None

**Question 66**

The sum of two numbers is 45 and the mean proportional between them is 18. The numbers are:

(a) (15, 30) (b) (32, 13) (c) (36, 9) (d) (25, 20)

**Question 67**

The sides of an equilateral triangle are shortened by 12 units 13 units and 14 units respectively and a right-angle triangle is formed. The side of the equilateral triangle is:

(a) 17 units (b) 16 units (c) 15 units (d) 18 units

**Question 68**

A distributor of apple Juice has 5000 bottles in the store that it wishes to distribute in a month. From experience it is known that demand $D$ (in number of bottles) is given by $D = -2000\,p^2 + 2000\,p + 17000$. The price per bottle that will result zero inventory is:

(a) ₹3 (b) ₹5 (c) ₹2 (d) None

**Question 69**

The sum of two irrational numbers multiplied by the larger one is 70 and their difference is multiplied by the smaller one is 12; the two numbers are:

(a) $3\sqrt{2}, 2\sqrt{3}$ (b) $5\sqrt{2}, 3\sqrt{5}$ (c) $2\sqrt{2}, 5\sqrt{2}$ (d) None

# Questions Based on Cubic Equations

**Question 70**

$x,\ x-4,\ x+5$ are the factors of the left-hand side of the equation:

(a) $x^3+2x^2-x-2=0$

(b) $x^3+x^2-20x=0$

(c) $x^3-3x^2-4x+12=0$

(d) $x^3-6x^2+11x-6=0$

**Question 71**

The equation $3x^3 + 5x^2 = 3x + 5$ has got 3 roots and hence the factors of the left-hand side of the equation $3x^3 + 5x^2 - 3x - 5 = 0$ are:

(a) $x-1,\ x-2,\ x-5/3$

(b) $x-1,\ x+1,\ 3x+5$

(c) $x+1,\ x-1,\ 3x-5$

(d) $x-1,\ x+1,\ x-2$

**Question 72**

The roots of $x^3 + x^2 - x - 1 = 0$ are:

(a) (–1, –1, 1) (b) (1, 1, –1) (c) (–1, –1, –1) (d) (1, 1, 1)

**Question 73**

If $4x^3+8x^2-x-2=0$, then the value of $(2x+3)$ is given by:

(a) 4, –1, 2 (b) –4, 2, 1 (c) 2, –4, –1 (d) None

**Question 74**

The value of $k$ is _____, if 2 is the root of the following cubic equation $x^3-(k+1)x+k=0$.

(a) 2 (b) 6 (c) 1 (d) 4

# Statistical Description of Data

**Question 1**

Which of the following statements is false?

(a) Statistics is derived from the Latin word ‘Status’
(b) Statistics is derived from the Italian word ‘Statista’
(c) Statistics is derived from the French word ‘Statistik’
(d) None of these

**Question 2**

Statistics is concerned with:

(a) Qualitative information
(b) Qualitative information
(c) (a) or (b)
(d) Both (a) and (b)

**Question 3**

Statistics is defined in terms of numerical data in the:

(a) Singular Sense
(b) Plural Sense
(c) Either (a) or (b)
(d) Both (a) and (b)

**Question 4**

Statistics is applied in:

(a) Economics
(b) Business Management
(c) Commerce and Industry
(d) All these

**Question 5**

An attribute is:

(a) A Qualitative Characteristic
(b) A Quantitative Characteristic
(c) A Measurable Characteristic
(d) All these

**Question 6**

Nationality of a student is:

(a) An attribute
(b) A continuous variable
(c) A discrete variable
(d) (a) or (c)

**Question 7**

Drinking habit of a person is:

(a) An attribute
(b) A variable
(c) A discrete variable
(d) A continuous variable

**Question 8**

Marks of a student is an example of

(a) An attribute
(b) A discrete variable
(c) A continuous variable
(d) None of these

**Question 9**

Annual income of a person is

(a) An attribute
(b) A discrete variable
(c) A continuous variable
(d) (a) or (c)

**Question 10**

Age of a person is

(a) An attribute
(b) A discrete variable
(c) A continuous variable
(d) A variable

**Question 11**

The data collected on the height of a group of students after recording their heights with a measuring tape are

(a) Primary Data
(b) Secondary Data
(c) Discrete Data
(d) Continuous Data

**Question 12**

The primary data are collected by

(a) Interview Method
(b) Observation Method
(c) Questionnaire Method
(d) All these

**Question 13**

The quickest method to collect primary data is

(a) Personal Interview
(b) Indirect Interview
(c) Telephone Interview
(d) By observation

**Question 14**

The best method to collect data, in case of a natural calamity, is

(a) Personal Interview
(b) Indirect Interview
(c) Questionnaire Method
(d) Direct Observation Method

**Question 15**

In case of a rail accident, the appropriate method of data collection is by:

(a) Personal Interview
(b) Direct Interview
(c) Indirect Interview
(d) All these

**Question 16**

Which method of data collection covers the widest area?

(a) Telephone Interview Method
(b) Mailed Questionnaire Method
(c) Direct Interview Method
(d) All these

**Question 17**

The amount of non-responses is maximum in

(a) Mailed Questionnaire Method
(b) Interview Method
(c) Observation Method
(d) All these

**Question 18**

Data collected on religion from the census reports are

(a) Primary Data
(b) Secondary Data
(c) Sample Data
(d) (a) or (b)

**Question 19**

Some important sources of secondary data are

(a) Some important sources of secondary data are
(b) International and primary sources
(c) Private and primary sources
(d) Government sources.

**Question 20**

Internal consistency of the collected data can be checked when

(a) Internal data are given
(b) External data are given
(c) Two or more series are given
(d) A number of related series are given

**Question 21**

The accuracy and consistency of data can be verified by:

(a) Internal checking
(b) External checking
(c) Scrutiny
(d) Both (a) and (b)

**Question 22**

The mode of presentation of data are

(a) Textual, tabulation and diagrammatic
(b) Tabular, internal and external
(c) Textual, tabular and internal
(d) Tabular, textual and external

**Question 23**

For tabulation, ‘caption’ is:

(a) The upper part of the table
(b) The lower part of the table
(c) The main part of the table
(d) The upper part of a table that describes the column and sub-column

**Question 24**

‘Stub’ of a table is the:

(a) Left part of the table describing the columns
(b) Right part of the table describing the columns
(c) Right part of the table describing the rows
(d) Left part of the table describing the rows

**Question 25**

The entire upper part of a table is known as

(a) Caption
(b) Stub
(c) Box Head
(d) Body

**Question 26**

The unit of measurement in tabulation is shown in

(a) Box Head
(b) Body
(c) Caption
(d) Stub

**Question 27**

In tabulation source of the data, if any, is shown in the

(a) Footnote
(b) Body
(c) Stub
(d) Caption

**Question 28**

Which of the following statements is untrue for tabulation?

(a) Statistical analysis of data requires tabulation
(b) It facilitates comparison between rows and not columns
(c) Complicated data can be presented
(d) Diagrammatic representation of data requires tabulation

**Question 29**

The most accurate mode of data presentation is

(a) Diagrammatic Method
(b) Tabulation
(c) Textual Presentation
(d) None of these

**Question 30**

The chart that uses logarithm of the variable is known as:

(a) Line Chart
(b) Ratio Chart
(c) Multiple Line Chart
(d) Component Line Chart

**Question 31**

Multiple line chart is applied for:

(a) Showing multiple charts
(b) Two or more related time series when the variables are expressed in the same unit
(c) Two or more related time series when the variables are expressed in different unit
(d) Multiple variations in the time series

**Question 32**

Multiple axis line chart is considered when:

(a) There is more than one time series
(b) The units of the variables are different
(c) (a) or (b)
(d) (a) and (b)

**Question 33**

Horizontal bar diagram is used for

(a) Qualitative Data
(b) Data varying over time
(c) Data varying over space
(d) (a) or (c)

**Question 34**

Vertical bar diagram is applicable when

(a) The data are qualitative
(b) The data are quantitative
(c) When the data vary over time
(d) (b) or (c)

**Question 35**

In order to compare two or more related series, we consider:

(a) Multiple Bar Chart
(b) Grouped Bar Chart
(c) (a) or (b)
(d) (a) and (b)

**Question 36**

Divided bar chart is considered for:

(a) Comparing different components of a variable
(b) The relation of different components to the table
(c) (a) or (b)
(d) (a) and (b)

**Question 37**

Pie-diagram is used for:

(a) Comparing different components and their relation to the total
(b) Representing qualitative data in a circle
(c) Representing quantitative data in circle
(d) (b) or (c)

**Question 38**

Cost of sugar in a month under the heads raw materials, labour, direct production and others were 12, 20, 35 and 23 units respectively. What is the difference between the central angles for the largest and smallest components of the cost of sugar?

(a) 72° (b) 48° (c) 56° (d) 92°

**Question 39**

Hidden trend, if any, in the data can be noticed in:

(a) Textual presentation
(b) Tabulation
(c) Diagrammatic Representation
(d) All these

**Question 40**

Diagrammatic representation of data is done by

(a) Diagrams
(b) Charts
(c) Pictures
(d) All these

**Question 41**

The best method of presentation of data is

(a) Textual
(b) Tabular
(c) Diagrammatic
(d) (b) and (c)

**Question 42**

The most attractive method of data presentation is

(a) Tabular
(b) Textual
(c) Diagrammatic
(d) (a) or (b)

**Question 43**

The distribution of shares is an example of the frequency distribution of:

(a) A discrete variable
(b) A continuous variable
(c) An attribute
(d) (a) or (c)

**Question 44**

The number of accidents for seven days in a locality are given below:

| No. of Accidents | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Frequency | 15 | 19 | 22 | 31 | 9 | 3 | 2 |

What is the number of cases when 3 or less accidents occurred?

(a) 56 (b) 6 (c) 68 (d) 87

**Question 45**

The distribution of profits of a blue-chip company relates to:

(a) A discrete variable
(b) A continuous variable
(c) An attribute
(d) (a) or (b)

**Question 46**

Mutually exclusive classification

(a) Excludes both the class limits
(b) Excludes the upper class limit but includes the lower class limit
(c) Includes the upper class limit but excludes the upper class limit
(d) Either (b) or (c)

**Question 47**

Mutually inclusive classification is usually meant for

(a) A discrete variable
(b) A continuous variable
(c) An attribute
(d) All these

**Question 48**

Mutually exclusive classification is usually meant for

(a) A discrete variable
(b) A continuous variable
(c) An attribute
(d) Any of these

**Question 49**

The LCB is

(a) An upper limit to LCL
(b) A lower limit to LCL
(c) (a) and (b)
(d) (a) or (b)

**Question 50**

The UCB is

(a) An upper limit to UCL
(b) A lower limit to LCL
(c) Both (a) and (b)
(d) (a) or (b)

**Question 51**

Length of a class is:

(a) The difference between the UCB and LCB of that class
(b) The difference between the UCL and LCL of that class
(c) (a) or (b)
(d) Both (a) and (b)

**Question 52**

For a particular class boundary, the less than cumulative frequency and more than cumulative frequency add up to:

(a) Total frequency
(b) Fifty per cent of the total frequency
(c) (a) or (b)
(d) None of these

**Question 53**

The following data relate to the incomes of 86 persons:

| Income in ₹ | 500 – 999 | 1000 – 1499 | 1500 – 1999 | 2000 – 2499 |
|---|---|---|---|---|
| No. of Persons: | 15 | 28 | 36 | 7 |

What is the percentage of persons earning more than ₹1,500?

(a) 50 (b) 45 (c) 40 (d) 60

**Question 54**

The following data relate to the marks of a group of students:

| Marks | Below 10 | Below 20 | Below 30 | Below 40 | Below 50 |
|---|---|---|---|---|---|
| No. of Students | 15 | 38 | 65 | 84 | 100 |

How many students got marks more than 30?

(a) 65 (b) 50 (c) 35 (d) 43

**Question 55**

Find the number of observations between 250 and 300 from the following data:

| Value | More than 200 | More than 250 | More than 300 | More than 350 |
|---|---|---|---|---|
| No. of Observations | 56 | 38 | 15 | 0 |

(a) 56 (b) 23 (c) 15 (d) 8

**Question 56**

A frequency distribution

(a) Arranges observations in an increasing order
(b) Arranges observation in terms of a number of groups
(c) Relates to a measurable characteristic
(d) All these

**Question 57**

The frequency distribution of a continuous variable is known as:

(a) Grouped Frequency Distribution
(b) Simple Frequency Distribution
(c) (a) or (b)
(d) (a) and (b)

**Question 58**

From the following data find the number class intervals if class length is given as 5.

73, 72, 65, 41, 54, 80, 50, 46, 49, 53

(a) 6 (b) 5 (c) 7 (d) 8

**Question 59**

Frequency density corresponding to a class interval is the ratio of:

(a) Class frequency to the total frequency
(b) Class frequency to the class length
(c) Class length to the class frequency
(d) Class frequency to the cumulative frequency

**Question 60**

Relative frequency for a particular class

(a) Lies between 0 and 1
(b) Lies between 0 and 1, both inclusive
(c) Lies between –1 and 0
(d) Lies between –1 to 1

**Question 61**

Mode of a distribution can be obtained from:

(a) Histogram
(b) Less than type Ogives
(c) More than type Ogives
(d) Frequency Polygon

**Question 62**

A comparison among the class frequencies is possible only in:

(a) Frequency Polygon
(b) Histogram
(c) Ogives
(d) (a) or (b)

**Question 63**

Frequency curve is a limiting form of

(a) Frequency Polygon
(b) Histogram
(c) (a) or (b)
(d) (a) and (b)

**Question 64**

Most of the commonly used frequency curves are

(a) Mixed
(b) Inverted J-shaped
(c) U-shaped
(d) Bell-shaped

**Question 65**

The distribution of profits of a company follows

(a) J-shaped frequency curve
(b) U-shaped frequency curve
(c) Bell-shaped frequency curve
(d) Any of these

**Question 66**

Median of a distribution can be obtained from

(a) Frequency Polygon
(b) Histogram
(c) Less than type Ogives
(d) None of these

**Question 67**

Out of 1000 persons, 25 per cent were industrial workers and the rest were agricultural workers. 300 persons enjoyed world cup matches on TV. 30 per cent of the people who had not watched world cup matches were industrial workers. What is the number of agricultural workers who had enjoyed world cup matches on TV?

(a) 260 (b) 240 (c) 230 (d) 250

**Question 68**

A sample study of the people of an area revealed that total number of women were 40% and the percentage of coffee drinkers were 45 as a whole and the percentage of male coffee drinkers was 20. What was the percentage of female non-coffee drinkers?

(a) 10 (b) 15 (c) 18 (d) 20