## Time Value of Money - FV, PV \& application

## Past Trends

| Attempt | SI \& CI | FV PV and <br> Other | Total |
| :---: | :---: | :---: | :---: |
| May 2018 | 3 | 3 | 6 |
| Nov 2018 | 11 | 3 | 14 |
| Jun 2019 | 7 | 3 | 10 |
| Nov 2019 | 10 | 3 | 10 |
| Nov 2020 | 7 | 7 | 14 |
| Jan 2021 | 10 | 4 | 14 |
| Jul 2021 | 6 | 7 | 13 |
| Dec 2021 | 4 | 3 | 7 |
| Jun 2022 | 2 | 8 | 10 |
| Dec 2022 | 8 | 6 | 14 |

## Types of Cashflows

| Single Cashflow | If single amount is paid or received initially and then direct finally at the end |  |
| :--- | :--- | :--- |
| Annuity | Annuity can be defined as a sequence of constant periodic payments (or <br> receipts) regularly over a specified period. |  |
| Types of Annuities | Annuity Regular | First payment/receipt at the end of the period |
|  | Annuity Due | First payment/receipt at the beginning of the period |

## Future Value

| Future Value - Single Cashflow | - Future value is the cash value of an investment at some time in the future. <br> - It is tomorrow's value of today's money compounded at the rate of interest. |
| :---: | :---: |
| Formula for FV of Single Cashflow | $\mathrm{FV}=\mathrm{CF}(1+\mathrm{i})^{\mathrm{n}}$ <br> where, CF = Single Cashflow for which FV is to be calculated, $\mathrm{i}=$ adjusted interest rate, $\mathrm{n}=\mathrm{no}$. of periods |
| FV of Annuity Regular | - To calculate final maturity value of an investment like RD where sum is invested in the annuity pattern starting at the end of each period. <br> - To calculate the final value of Sinking Fund or Savings amount to achieve the target maturity value. |
| Formula for Future Value - Annuity Regular | $\begin{aligned} & \text { FVAR }=A_{i} \times F \operatorname{VAF}(n, i) \\ & \text { FVAR }=A_{i} \times\left\{\frac{\left[(1+i)^{n}-1\right]}{i}\right\} \end{aligned}$ <br> where, FVAR = Future Value of Annuity Regular, $\mathbf{A}_{\mathbf{i}}=$ Annuity Value (Installment), FVAF = Future Value Annuity Factor, $\mathbf{i}=$ adjusted interest rate, $\mathbf{n}=$ no. of periods |


| FV of Annuity Due | - To calculate final maturity value of an investment like RD where sum is invested in the annuity pattern at the beginning of each period <br> - To calculate final maturity value of an investment like RD where sum is invested in the annuity pattern at the beginning of each period |
| :---: | :---: |
| Formula for Future Value - Annuity Due | $\begin{aligned} & \text { FVAD }=A_{i} \times \operatorname{FVAF}(n, i) \times(1+i) \\ & \text { FVAD }=A_{i} \times\left\{\frac{\left[(1+i)^{n}-1\right]}{i}\right\} \times(1+i) \end{aligned}$ <br> where, FVAD = Future Value of Annuity Due, $\boldsymbol{A}_{\mathbf{i}}=$ Annuity Value (Installment), FVAF = Future Value Annuity Factor, $\mathbf{i}=$ adjusted interest rate, $\mathbf{n}=$ no. of periods |
| Sinking Fund | - It is the fund credited for a specified purpose by way of sequence of periodic payments over a time-period at a specified interest rate. <br> - Interest is compounded at the end of every period. <br> - Size of the sinking fund deposit is same as Future Value of Annuity |
| Compounding and Discounting | Compounding <br> (Adding the interest) $\times(1+i)^{n}$ |
|  | Discounting <br> (Removing the interest) $\times \frac{1}{(1+i)^{n}}$ |

## Present Value

| Present Value of Single Cashflow | - Present value is today's value of tomorrow's money discounted at the interest rate |
| :---: | :---: |
| Formula for PV of Single Cashflow | $\mathrm{PV}=\frac{\mathrm{CF}}{(1+\mathrm{i})^{n}}$ <br> where, CF = Single Cashflow for which PV is to be calculated, $\mathrm{i}=$ adjusted interest rate, $n=$ no. of periods |
| Present Value - <br> Annuity Regular | Use: To calculate loan amount when periodic installments value are given and vice-versa. <br> Application: Leasing, Capital Expenditure etc. |
| Formula for PV of Annuity Regular | $\begin{gathered} \operatorname{PVAR}=A_{i} \times \operatorname{PVAF}(n, i) \\ \operatorname{PVAR}=A_{i} \times\left[\frac{1}{i} \times\left\{1-\frac{1}{(1+i)^{n}}\right\}\right] \end{gathered}$ <br> where, PVAR = Present Value of Annuity Regular, $\boldsymbol{A}_{\mathbf{i}}=$ Annuity Value (Installment), PVAF = Present Value Annuity Factor, $\mathbf{i}=$ adjusted interest rate, $\mathbf{n}=$ no. of periods |
| Calculator Trick for PVAF | $1+\mathrm{i}) \div=\ldots . . \mathrm{n}-$ times GT |
| Formula for Present Value of Annuity Due | $\operatorname{PVAD}=\left[A_{i} \times \operatorname{PVAF}\{(\mathrm{n}-1), \mathrm{i}\}\right]+\mathrm{A}_{\mathrm{i}}$ |

## Applications of TVOM \& Other Concepts

| Leasing | - Lessor: Owner of Asset, who gives asset on rent. Lease Rentals are income for Lessor <br> - Lessee: User of the asset who has taken asset on rent. Lease Rentals are expense for Lessee <br> - Use of TVOM: Present Value of Annuity (Lease Rentals) are compared with asset cash down price to decide if leasing is preferable or not. |  |  |
| :---: | :---: | :---: | :---: |
| Capital Expenditure Decisions | - Present value of future benefits due to new asset are compared with purchase value of asset, to decide whether asset to purchase or not. |  |  |
|  | - Present value of interest income and maturity value is compared with the issue price of bond <br> - Terms |  |  |
|  | Bond | It is a debt security. Type of loan taken by company from public. Like debentures |  |
| Valuation of Bond | Face Value/ Par Value | Value written on the document of bond. This value is used to calculate Interest Amount |  |
|  | Issue Price | Actual payment made to purchase the bond |  |
|  | Maturity Value | Amount to be received on redemption or maturity of bond |  |
| PV of Perpetuity | $\mathrm{PVP}=\frac{A_{i}}{i}$ <br> where, PVP = Present Value of Perpetuity, $\mathbf{A}_{\mathbf{i}}=$ Annuity Value (Installment), $\mathbf{i}=$ adjusted interest rate |  |  |
| PV Growing <br> Perpetuity | A stream of cashflows that grows at constant rate forever is known as growing perpetuity. $\text { PVGP }=\frac{A_{i}}{i-g}$ <br> where, <br> PVGP = Present Value of Growing Perpetuity <br> $\mathbf{A}_{\mathbf{i}}=$ Annuity Value (Installment) <br> $\mathbf{i}=$ adjusted interest rate <br> g = growth rate |  |  |
| Net Present Value | Formula |  | NPV = Present Value of Cash Inflows <br> - Present Value of Cash Outflows |
|  |  | n Base | If NPV $\geq \mathbf{0}$, accept the proposal, If NPV $\mathbf{~ 0}$, reject the proposal |
| Real Rate of Return | Real Rat | Return | Nominal Rate of Return - Rate of Inflation |
| CAGR | Compounded | ual Grow | th rate used to show annual growth as per Cl |

## MCQs

A person invests ₹ 500 at the end of each year with a bank which pays interest at $10 \%$ p.a. C.I. annually. The amount standing to his credit one year after he has made his yearly

Exercise investment for the $12^{\text {th }}$ time is.
a. 11,761.36
b. 10,692.34
c. 12,000
d. None

Ans: a

Exercise

Ans: c

A person bought a house paying ₹ 20,000 cash down and ₹ 4,000 at the end of each year for 25 yrs. at $5 \%$ p.a. C.I. The cash down price is
a. 75,000
b. 76,000
c. $76,375.5$
d. None

Johnson left ₹ 1,00,000 with the direction that it should be divided in such a way that his minor sons Tom, Dick and Harry aged 9, 12 and 15 years should each receive equally after attaining the age 25 years. The rate of interest being $3.5 \%$, how much each son receives after getting 25 years old?
a. 50,000
b. 51,947
c. 52,000
d. None

Ans: b

## Exercise

Ans: b

## Exercise

Ans: a

## Exercise

Ans: c

## Exercise

Ans: b
PYQ
June 19

Ans: b

## PYQ

 June19A sinking fund is created for redeeming debentures worth ₹ 5 lakhs at the end of 25 years. How much provision needs to be made from profits each year provided sinking fund investments can earn interest at $4 \%$ p.a.?
a. 50,000
b. 51,994
c. 52,000
d. None

The amount of an annuity certain of $₹ 150$ for 12 years at $3.5 \%$ p.a C.I is
a. 2190.28
b. $\quad 1290.28$
c. 2180.28
d. None

A loan of ₹ 10,000 is to be paid back in 30 equal instalments. The amount of each installment to cover the principal and at $4 \%$ p.a Cl is
a. 587.87
b. 587
c. 578.30
d. None

Appu retires at 60 years receiving a pension of 14,400 a year paid in half-yearly installments for rest of his life after reckoning his life expectation to be 13 years and that interest at $4 \%$ p.a. is payable half-yearly. What single sum is equivalent to his pension?
a. 1,45,000
b. 1,44,900
c. $1,44,800$
d. 1,44,700

A person wants to lease out a machine costing ₹5,00,000 for a 10-year period. It has fixed rental of ₹51,272 per annum payable annually starting from the end of the first year. Suppose rate of interest is $10 \%$ p.a. compounded annually. To whom this agreement is favorable?
a. Favor of Lessee
b. Favor of Lessor
c. Not for both
d. Can't be determined

Determine the present value of perpetuity of ₹ 50,000 per month @ rate of interest $12 \%$ p.a. is $\qquad$
a. $45,00,000$
b. $50,00,000$
c. $55,00,000$
d. $60,00,000$

Ans: b

## PYQ

Jul 21
Ans: a

## PYQ

Jul 21
Ans: a

## PYQ

Jul 21

| Years | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Operating Profit | 90 | 100 | 106.4 | 107.14 | 120.24 | 157.34 |

Find CAGR
a. 9\%
b. $12 \%$
c. $11 \%$
d. $13 \%$

Ans: b
If the cost of capital is $12 \%$ p.a., then the NPV from the given cashflow is

| Years | 0 | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- | :--- |
| Cashflow | $(100)$ | 60 | 40 | 50 |

a. 31048
b. 34185
c. 21048
d. 24187

Ans: c
An investor intends purchasing a three-year Rs. 1000 par value bond having nominal interest rate of $10 \%$. At what price the bond may be purchased now if it matures at par and the investor requires a rate of return of $14 \%$ ?
a. 907.125
b. 900.36
c. 916.66
d. 569.22

Ans: a
Let a person invest a fixed sum at the end of each month in an account paying interest

PYQ
Jun 19 $12 \%$ per year compounded monthly. If the future value of this annuity after the $12^{\text {th }}$ payment is ₹ 55,000 then the amount invested every month is?
a. ₹ 4,837
b. ₹ 4,637
c. ₹ 4,337
d. ₹ 3,337

Ans: c

## PYQ

 value of the perpetuity if the rate of return is $20 \%$ ?a. 20.1
b. 19.1
c. 21.1
d. 22.1

Ans: a
The present value of an annuity immediate is the same as
a. Annuity regular for ( $n-1$ ) year plus the initial receipt in the beginning
b. Annuity regular for $(n-1)$ years
c. Annuity regular for $(n+1)$ years
d. Annuity regular for $(n+1)$ year plus the initial receipt in the beginning

Ans: a

PYQ
Dec 22

## Ans: a

10 years ago the earning per share (EPS) of ABC Ltd. was ₹ 5 share. Its EPS for this year is ₹ 22 . Compute at what rat, EPS of the company grow annually?
a. $15.97 \%$
b. $16.77 \%$
c. $18.64 \%$
d. $14.79 \%$

Sinking fund factor is the reciprocal of:
a. Present value interest factor of a single cash flow
b. Present value interest factor of an annuity
c. Future value interest factor of an annuity
d. Future value interest factor of a single cash flow

Ans: c

