Past Trends

Attempt	Easy	Moderate	Advance Level	Total
May 2018	**4**	**1**	**1**	**6**
Nov 2018	**4**	**1**	**0**	**5**
Jun 2019	**4**	**1**	**0**	**5**
Nov 2019	**4**	**2**	**0**	**6**
Nov 2020	**5**	**1**	**1 (o/s)**	**7**
Jan 2021	**3**	**0**	**1 (o/s)**	**4**
Jul 2021	**3**	**2**	**0**	**5**
Dec 2021	**3**	**2**	**1**	**6**
Jun 2022	**3**	**0**	**0**	**3**
Dec 2022	**3**	**1**	**0**	**4**

Modern Probability

Approaches of Probability	• Classical Probability • Set Based Probability • Modern Probability (Axiomatic)
Modern Probability	• Unlike other, here probability is not calculated by a simple ratio • Probability is calculated by using a function defined on random variable
Probability Function	Probability = f(X) where X is a random variable Types of Probability Function **Discrete**: Probability Mass Function; **Continuous**: Probability Density Function
Parameter	characteristics that are used to define a given population

Binomial Distribution

When to use	• When random variable is discrete • Probability is not very low (like 1/2, 2/3, 1/4 around)
Binomial Variate	$X \sim B(n,p)$ Number of Parameters: 2 Binomial Distribution: **bi-parametric discrete** probability distribution
Probability Mass Function	$f(x) = P(X = x) = {}^{n}C_{x} p^{x} q^{n-x}$ X can take whole number value from **0 to n**
Mean	$\mu = np$
Variance	$\sigma^2 = npq$ • Mean is always more than variance • Maximum Value of Variance $= \frac{n}{4}$

<table>
<tr><td>Mode</td><td>
<table>
<tr><td colspan="2">Find $(n+1)p$</td></tr>
<tr><td>If result is integer, then there are two modes. $\mu_0=(n+1)p$ $\mu_0=(n+1)p-1$</td><td>If result is non-integer, then there is only one mode

Largest integer contained in $(n+1)p$</td></tr>
</table>
</td></tr>
<tr><td>Additive Property</td><td>If $X\sim B(n_1,p)$ and $Y\sim B(n_2,p)$ then $X+Y\sim B(n_1+n_2,p)$</td></tr>
</table>

PYQ May18 The variance of binomial distribution with parameters n and p is

a. $np^2(1-p)$ b. $\sqrt{np(1-p)}$ c. $nq(1-q)$ d. $n^2q^2(1-q)$

Ans: c

PYQ May 18 An example of bi-parametric discrete probability distribution

a. Binomial b. Poisson c. Normal d. a and b

Ans: a

PYQ May 18 Probability Distribution may be

a. discrete b. continuous c. infinite d. a or b

Ans: d

PYQ Nov 18 The mean of binomial distribution $X\sim B(4,\frac{1}{3})$ is equal to

a. 3/5 b. 8/3 c. 3/4 d. 4/3

Ans: d

PYQ Nov18 The probability that a student is not a swimmer is 1/5, then the probability that out of five students four are swimmer is

a. $\left(\frac{4}{5}\right)^4\frac{1}{5}$ b. ${}^5C_1\left(\frac{1}{5}\right)^4\frac{4}{5}$ c. ${}^5C_4\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)^1$ d. None

Ans: c

PYQ June 19 If mean and variance are 5 and 3 respectively then the relation between p and q is

a. $p>q$ b. $p<q$ c. $p=q$ d. p is symmetric

Ans: b

PYQ Nov 19 Find mode when n = 15 and p = $\frac{1}{4}$ in binomial distribution?

a. 4 b. 4 and 3 c. 4.2 d. 3.75

Ans: b

PYQ Jul 21 If x is a binomial variate with p = $\frac{1}{3}$ for the experiment of 90 trials, then the SD is equal to

a. $\sqrt{5}$ b. $-\sqrt{5}$ c. $2\sqrt{5}$ d. $\sqrt{15}$

Ans: c

PYQ Dec 21

Four unbiased coins are tossed simultaneously. The expected number of heads is

a. 1 b. 2 c. 3 d. 4

Ans: b

PYQ Dec 22

Standard Deviation of Normal Distribution is

a. npq b. $\sqrt{npq}$ c. np d. $\sqrt{np}$

Ans: b

Poisson Distribution

When to use	• When random variable is discrete • Probability is very low (like 1%, 2% around)
Poisson Parameter	• Poisson distribution has only one parameter • Denoted by m • It can either be given in question or can be calculated using formula $m = np$
Poisson Variate	$X \sim P(m)$
Probability Mass Function	$f(x) = P(X = x) = \frac{e^{-m}.m^x}{x!}$ X can take whole number value from **0 to n**
Mean	$\mu = m$
Variance	$\sigma^2 = m$
Mode	Find **m** If result is integer, then there are two modes. $\mu_0 = m$, $\mu_0 = m - 1$ If result is non-integer, then there is only one mode. **Largest integer** contained in **m**
Additive Property	If $X \sim P(m_1)$ and $Y \sim P(m_2)$ then $X + Y \sim P(m_1 + m_2)$

PYQ Nov 18
PYQ Jun 19
PYQ Nov 19
PYQ Nov 20
PYQ Dec 21

For a Poisson variate X, P(X = 2) = 3P(X = 4), then the SD of X is

a. 2 b. 4 c. $\sqrt{2}$ d. 3

Ans: c

PYQ Nov 19

For a Poisson distribution

a. Mean and SD are equal
b. Mean and Variance are equal
c. SD and Variance are equal
d. Both a and b

Ans: b

PYQ Nov 20
PYQ Jan 21

Which of the following is uni-parametric distribution?

a. Poisson b. Normal c. Binomial d. Chi

Ans: a

PYQ Nov 20

If the parameter of Poisson distribution is m and (Mean + SD) = 6/25 then find m

a. 3/25 b. 1/25 c. 4/25 d. 3/5

Ans: b

PYQ Jan 21

If x is a Poisson Variable and P(X=1) = P(X=2), then P(X=4) is

a. $\frac{2}{3}e^{-2}$ b. $\frac{2}{3}e^{4}$ c. $\frac{3}{2}e^{-2}$ d. None

Ans: a

PYQ Dec 21

The average number of advertisements per page appearing in a newspaper is 3. What is the probability that in a particular page zero number of advertisements are there?

a. e^{-3} b. e^{0} c. e^{3} d. e^{-1}

Ans: a

PYQ Dec 21

The average number of advertisements per page appearing in a newspaper is 3. What is the probability that in a particular page zero number of advertisements are there?

a. e^{-3} b. e^{0} c. e^{3} d. e^{-1}

Ans: a

PYQ Jun 22

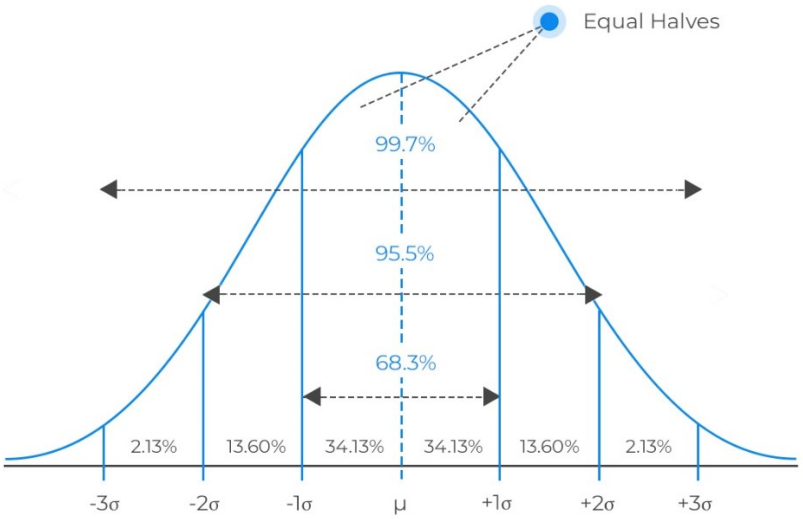
If SD of Poisson Variable is 1.732 then what is the value of $P(-2.48 < X < 3.54)$ is

a. 0.73 b. 0.65 c. 0.86 d. 0.81

Ans: b

Normal Distribution

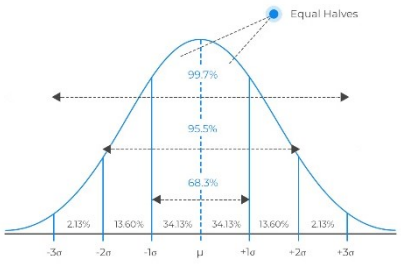
When to use	• When random variable is continuous • It is bi-parametric continuous probability distribution
Normal Variable	$X \sim N(\mu, \sigma^2)$
Special Point about Mean	Mean = Median = Mode = μ (symmetric distribution)
Variance	Here σ^2 is a parameter and will be given in question
Mean Deviation	0.8σ
Quartiles	$Q_1 = \mu - 0.675\sigma$ $Q_3 = \mu + 0.675\sigma$
Quartile Deviation	$QD = 0.675\sigma$
Ratio between QD:MD:SD	10:12:15
Probability Density Function	$\frac{1}{\sigma\sqrt{2\pi}} e^{-(\frac{x-\mu}{\sigma})^2 \times \frac{1}{2}}$ This is used only for the purpose of obtaining mean and SD comparing the function given in question with this function.



Normal Curve	Curve formed by normal distribution. It is also called as Probability Curve
Shape of Normal Curve	Bell Shaped
Points of Inflexion	$\mu-\sigma$ and $\mu+\sigma$
Area under Normal Curve	The total area of the normal curve or for that any probability curve is taken to be unity i.e. one.
Tails of Normal Curve	the two tails of the normal curve extend indefinitely on both sides of the curve and both the left and right tails never touch the horizontal axis.
Symmetric Distribution	The line drawn through $x=\mu$ has divided the normal curve into two parts which are equal in all respect.
Popular Probability Intervals in Normal Curve	

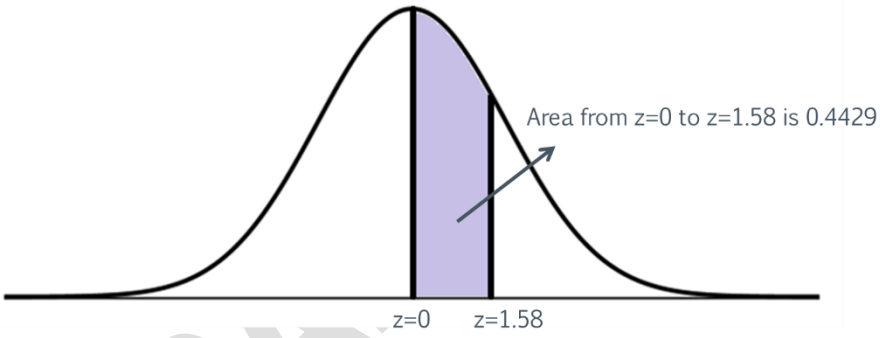
From	To	Area/ Probability
μ	$\mu+\sigma$	34.135%
$\mu+\sigma$	$\mu+2\sigma$	13.59%
$\mu+2\sigma$	$\mu+3\sigma$	2.14%
$\mu+3\sigma$	∞	0.135%

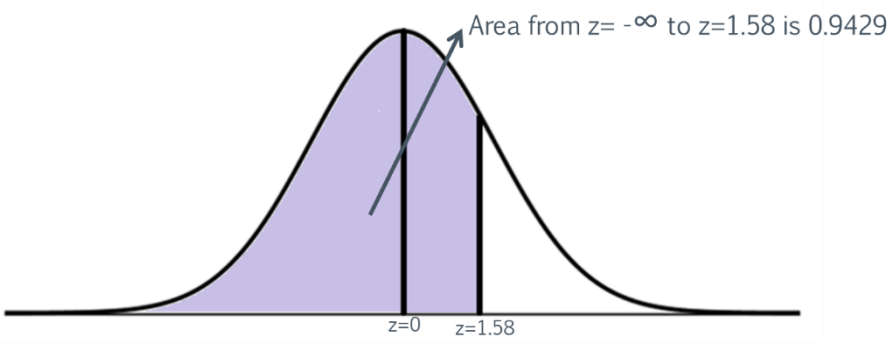
From	To	Area/ Probability
$\mu-\sigma$	$\mu+\sigma$	68.27%
$\mu-2\sigma$	$\mu+2\sigma$	95.45%
$\mu-3\sigma$	$\mu+3\sigma$	99.73%



Standard Normal Distribution

When to use	• When probability calculation is outside the popular intervals • Normal Distribution is converted to Standard Normal Distribution so that Z Table Table can be used for probability calculation
Conditions	Normal Distribution where $\mu = 0$ and $\sigma = 0$
Convert X to Z	$Z = \frac{X - \mu}{\sigma}$
μ	Mean = Median = Mode = 0
Variance	$\sigma^2 = 1$ and $\sigma = 1$
Points of Inflexion	-1 and +1
Mean Deviation	0.8
Quartile Deviation	0.675
Use of Z Table	This table gives us the probability of values from z = 0 to any value of z. (Area from center)
Cumulative Distribution Function	$\varphi(k) = P(X \leq k)$ $\varphi(k)$ gives the area from $-\infty$ to point k in a standard distribution





PYQ May 18 If the area of standard normal curve between z = 0 to z = 1 is 0.3413, then the value of $\varphi(1)$ is

a. 0.5 b. 0.8413 c. -0.5 d. 1

Ans: b

PYQ May 18
PYQ Nov 18

What is the first quartile of x having the following probability density function:

$$f(x)=\frac{1}{\sqrt{72\pi}}e^{\frac{-(x-10)^2}{72}}$$

a. 4 b. 5 c. 5.95 d. 6.75

Ans: c

PYQ Nov 18

If for a normal distribution $Q_1=54.52$ and $Q_3=78.86$, then the median of the distribution is

a. 12.17 b. 39.43 c. 66.69 d. None

Ans: c

PYQ Jun 19

Area between $\mu-1.96\sigma$ and $\mu+1.96\sigma$ is

a. 95.45% b. 95% c. 96% d. 99%

Ans: b

PYQ Jun 19

If points of inflexion of a normal curve are 40 and 60 respectively, then mean deviation is

a. 8 b. 45 c. 50 d. 60

Ans: a

PYQ Nov 19

Area under $\mu\pm3\sigma$

a. 99.73% b. 99% c. 100% d. 99.37%

Ans: a

PYQ Nov 20

If we change the parameter of a _____ distribution the shape of probability curve does not change.

a. Normal b. Binomial c. Poisson d. None

Ans: a

PYQ Nov 20

The quartile deviation of a normal distribution with mean 10 and standard deviation 4 is _____

a. 54.24 b. 23.20 c. 0.275 d. 2.70

Ans: d

PYQ Jul 21

In normal distribution Mean, Median and Mode are

a. Zero b. Equal c. Not Equal d. Can't Say

Ans: b

PYQ Jun 22

In a normal distribution, variance is 16 then the value of mean deviation is

a. 4.2 b. 3.2 c. 4.5 d. 2.5

Ans: b