Probability | Past Trends

Attempt	Easy	Moderate	Advance Level	Total
May 2018	**6**	**1**	**0**	**7**
Nov 2018	**3**	**3**	**0**	**6**
Jun 2019	**3**	**1**	**1 (Bayes')**	**5**
Nov 2019	**4**	**1**	**0**	**5**
Nov 2020	**3**	**1**	**0**	**4**
Jan 2021	**5**	**1**	**0**	**6**
Jul 2021	**4**	**3**	**0**	**7**
Dec 2021	**4**	**2**	**1**	**7**
Jun 2022	**5**	**1**	**1**	**7**
Dec 2022	**6**	**1**	**0**	**7**

Probability - Basics

History

- first application of probability was made by a group of mathematicians in Europe
- about three hundred years back
- to enhance their chances of winning
- in different games of **gambling**

Types

- **Subjective Probability**
  - personal judgement
  - experience
  - influenced by the **personal belief, attitude, and bias**
  - of the person applying it
- **Objective Probability**
  - Based on **Rules and Maths**
  - This is what we are going to discuss

Random Experiment

Experiment	A performance that produces certain results
Random Experiment	An experiment is defined to be random if the results of the experiment depend **on chance** only.
Examples	Tossing a coin, throwing a dice, drawing cards from a pack

Events

The results or outcomes of a **random experiment** are known as **Events**

Event at its simplest form	Simple Event or Elementary Event
Event that can be sub-divided into further events	Composite Event or Compound Event

Classical Probability

General

- Also called Prior Definition of Probability, this formula is Event (Result) Based.
- It is given by Bernoulli and Laplace.

Formula of Probability

$$P(A) = \frac{\text{no. of favorable outcomes}}{\text{total number of outcomes}}$$

<table>
<tr><td>Formula to obtain total outcomes</td><td>Total number of possible outcomes: p^q
where, p = no. of events in one trial, q = no. of trials</td></tr>
<tr><td>Conditions</td><td>• Value of Probability: $0 \le P(A) \le 1$
• If $P(A)=1$, A is a sure event
• If $P(A)=0$, A is an impossible event</td></tr>
<tr><td>Complimentary Probability</td><td>• Probability of non-occurrence of an event A is denoted by $P(A')$ or $P(\bar{A})$ is called as complimentary event of A.
$$P(\bar{A}) = 1 - P(A)$$</td></tr>
<tr><td>Odds in Favour of an Event A</td><td>$$\frac{\text{number of favorable events to A}}{\text{number of unfavorable events to A}}$$</td></tr>
<tr><td>Odds against an Event A</td><td>$$\frac{\text{number of unfavorable events to A}}{\text{number of favorable events to A}}$$</td></tr>
<tr><td>Playing Cards</td><td>
<table>
<tr><td>Suites</td><td>Spades</td><td>Hearts</td><td>Diamond</td><td>Clubs</td></tr>
<tr><td>Symbol</td><td>♠</td><td>♡</td><td>♢</td><td>♣</td></tr>
<tr><td>Count</td><td>13</td><td>13</td><td>13</td><td>13</td></tr>
</table>
<table>
<tr><td>Ranks</td><td>Ace</td><td>King</td><td>Queen</td><td>Jack</td><td>Numbers</td></tr>
<tr><td>Symbol</td><td>A</td><td>K</td><td>Q</td><td>J</td><td>10, 9, 8, 7, 6, 5, 4, 3, 2</td></tr>
<tr><td>Count</td><td>4</td><td>4</td><td>4</td><td>4</td><td>4</td></tr>
</table>
</td></tr>
<tr><td>Limitations</td><td>• Applicable only when events are finite and are equally likely
• Limited application of this definition like in tossing coin, throwing dice, cards etc.</td></tr>
</table>

PYQ May 18

Two broad divisions of probability are
a. Subjective, Mathematical
b. Deductive, Mathematical
c. Subjective, Objective
d. Deductive, Objective

Ans: c

PYQ May 18

The term chance and probability are synonyms
a. True b. False c. Both d. None

Ans: a

PYQ Nov 18

Two dices are thrown simultaneously, then the probability that the sum of two numbers appearing on the top of dice is 9 is
a. 8/9 b. 1/9 c. 7/9 d. None

Ans: b

PYQ Jun 19

If a coin is tossed 5 times, then the probability of getting Tail and Head occurs **alternatively**
a. 1/8 b. 1/16 c. 1/32 d. 1/64

Ans: b

PYQ Jun 19
PYQ Jan 21
PYQ Jun 22

Two dices are thrown simultaneously, then the probability of getting at least one 5 is
a. 11/36 b. 5/36 c. 8/15 d. 1/7

Ans: a

PYQ Nov 19

A bag contains 15 one-rupee coins, 25 two rupee coins and 10 five rupee coins if a coin is selected at random than probability for not selecting a one rupee coin is:

a. 0.30 b. 0.20 c. 0.25 d. 0.70

Ans: d

PYQ Nov 20

When 2 fair dice are thrown what is the probability of getting a sum which is multiple of 3?

a. 4/36 b. 13/36 c. 2/36 d. 12/36

Ans: d

PYQ Nov 20
PYQ Jan 21

When two coins are tossed simultaneously the probability of getting at least one tail?

a. 1 b. 0.75 c. 0.5 d. 0.25

Ans: b

PYQ Jan 21

An event that can be sub-divided into further events is called as

a. Composite b. Complex c. Mixed d. Simple

Ans: a

PYQ Jan 21

Three identical and balanced dice are rolled. The probability that the same number will appear on each of them is

a. 1/6 b. 1/18 c. 1/36 d. 1/24

Ans: c

PYQ Jan 21

If an unbiased coin is tossed three times, what is the probability of getting more than one head?

a. 1/2 b. 3/8 c. 7/8 d. 1/3

Ans: a

PYQ Jun 22

A dice is rolled twice. Find the probability of getting numbers multiple of 3 or 5?

a. 1/3 b. 1/4 c. 1/2 d. 1/6

Ans: c

PYQ Jun 22

If two dice are rolled and one of the dice shows 1 at a point then how many such outcome can be done where it is known that its probability is $\frac{x}{36}$ where x = ____

a. 11 b. 7 c. 8 d. 9

Ans: a

PYQ Jun 22

If $\frac{p}{q}$ are odds in favor of an event, then probability of that event is

a. $\frac{p}{q}$ b. $\frac{p}{p+q}$ c. $\frac{q}{p+q}$ d. $\frac{q}{p}$

Ans: b

Usage of Combinations in Probability

Criteria	If only one object is to be selected	No need to use combination
	If more than one object is to be selected	Use combinations to calculate both favorable outcome and total outcome

PYQ Nov 19

Two letters are chosen from the word HOME. What is the probability that the letters chosen are not vowels.

a. 1/2 b. 1/6 c. 2/3 d. 0

Ans: b

PYQ Nov 19

A basket contains 15 white balls, 25 red balls and 10 blue balls. If a ball is selected at random, the probability of selecting not a white ball.

a. 0.20 b. 0.25 c. 0.60 d. 0.70

Ans: d

PYQ Jul 21

If there are 16 phones, 10 of them are Android and 6 of them are of Apple IOS, then the probability of 4 randomly selected phones to include 2 Android and 2 Apple phone is

a. 0.47 b. 0.51 c. 0.37 d. 0.27

Ans: c

Set Based Probability

Terms	**Term in Probability**	**Term in Sets**	**Explanation**
	Sample Space	Universal Set	Set all the possible events for a random experiment
	Sample Points	Elements	Each event of a Random experiment is termed as Sample Point
	Event Set A	Ordinary Set A which is a subset of Universal Set	Event Set A which is under consideration for probability calculations is defined as a non-empty subset of Set S (Sample Space) containing all favorable sample points for A
Formula of Probability Event A	$P(A)=\frac{\text{number of sample points is Set A}}{\text{number of sample points is Sample Space S}}$ $P(A)=\frac{n(A)}{n(S)}$		
Why Sets?	• To use concept of **union, intersection** in probability • To use Sets formula for **complicated Probability Questions**		
2-Sets Formula	$n(A\cup B)=n(A)+n(B)-n(A\cap B)$		
3-Sets Formula	$n(A\cup B\cup C)=n(A)+n(B)+n(C)-n(A\cap B)-n(B\cap C)-n(C\cap A)+n(A\cap B\cap C)$		

Types of Sets of Events

Mutually Exclusive or Incompatible Events	Events are exclusive if they **cannot occur simultaneously**
Exhaustive Events	Events are exhaustive if **one of them must necessarily occur**
Equally likely or Equi-Probable Events	Events are equally likely if all of them **have same probability** (also called mutually symmetric events)

Addition Theorem of Probability

Theorem	Formula	Applicable for
1	$P(A \cup B) = P(A) + P(B)$	For two mutually exclusive events
2	$P(A_1 \cup A_2 \cup A_3 \cup ...) = P(A_1) + P(A_2) + P(A_3) + ...$	For any number of mutually exclusive events
3	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	For any two events
4	$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$	For any three events

Expected Frequency of occurrence of an event

Expected Frequency of Event A = P(A) × N (total number of outcomes)

Probability of A – B or B – A

Probability of **Only A**	$P(A-B)$	$P(A \cap B')$	$P(A) - P(A \cap B)$
Probability of **Only B**	$P(B-A)$	$P(A' \cap B)$	$P(B) - P(A \cap B)$

PYQ May 18

Sum of all probabilities mutually exclusive and exhaustive events is equal to

a. 0 b. 1/2 c. 1/4 d. 1

Ans: d

PYQ Nov 18

If $P(A) = 1/2$, $P(B) = 1/3$, $P(A \cap B) = 1/4$, then $P(A \cup B)$ is

a. 11/12 b. 10/12 c. 7/12 d. 1/6

Ans: c

PYQ Nov 18
PYQ Jun 22

The probability that a leap year has 53 Wednesdays is

a. 2/7 b. 3/5 c. 2/3 d. 1/7

Ans: a

PYQ Nov 18

If $P(A \cup B) = 0.8$ and $P(A \cap B) = 0.3$, then $P(A') + P(B')$ is equal to

a. 0.3 b. 0.5 c. 0.7 d. 0.9

Ans: d

PYQ Nov 19

If A, B and C are three mutually exclusive and exhaustive events such that P(A) = 2P(B) = 3P(C), what is P(B)?

a. 6/11 b. 3/11 c. 1/6 d. 1/3

Ans: b

Conditional Probability

Dependent Events	If occurrence of one event is **influenced** by occurrence of another event, then two events are dependent. Conditional Probability is applicable only for dependent events
Independent Events	Two events are said to be independent if occurrence of one event do not influence the occurrence of other. Conditional Probability is **not applicable**
Formula for Conditional Probability	Conditional Probability event B given that event A has already been occurred: $P(B/A)=\frac{P(B\cap A)}{P(A)}$ where $P(A)\neq 0$ Conditional Probability event B given that event A has already been occurred: $P(A/B)=\frac{P(A\cap B)}{P(B)}$ where $P(B)\neq 0$
Compound Theorem for Dependent Events	$P(A\cap B)=P(A/B)\times P(B)$ $P(A\cap B)=P(B/A)\times P(A)$ Joint Probability = Unconditional Probability of one event × Conditional Probability of another event
Compound Theorem for Independent Events	In case of independent events, Conditional and Unconditional are same $P(A\cap B)=P(A)\times P(B)$ If A and B are independent events, then their complementary events are also independent and same theorem can be applied $P(A'\cap B)=P(A')\times P(B)$ $P(A\cap B')=P(A)\times P(B')$ $P(A'\cap B')=P(A')\times P(B')$
De-Morgan's Law Application	$P(A\cup B\cup C)=1-P(A\cup B\cup C)'$ $P(A\cup B\cup C)=1-P(A'\cap B'\cap C')$ $P(A\cup B\cup C)=1-P(A')\times P(B')\times P(C')$

PYQ May 18

The theorem of compound probability states that for any two events A and B is

a. $P(A\cap B)=P(A/B)\times P(B)$
b. $P(A\cup B)=P(A/B)\times P(B)$
c. $P(A\cap B)=P(A)\times P(B)$
d. None

Ans: a

PYQ Nov 18

Ram is known to hit a target in 2 out of 3 shots whereas Shyam is known to hit the same target in 5 out of 11 shots. What is the probability that target would hit if they both try?

a. 9/11 b. 3/11 c. 10/33 d. 6/11

Ans: a

PYQ Nov 20

If A speaks 75% truth and B speaks 80% truth. In what % both are likely contradict each other in narrating the same question?

a. 0.60 b. 0.45 c. 0.65 d. 0.35

Ans: b

PYQ Jul 21

The probability that a football team losing a match at Kolkata is 3/5 and winning a match at Bengaluru is 6/7, the probability of the team winning at least one match is _____

a. 3/35 b. 18/35 c. 32/35 d. 17/35

Ans: c

Random Variable | Probability Distribution

Random Variable	It is a variable defined on Sample Space of a random experiment that can take any value (Real Number)
Probability Distribution	It is defined as the statement/ table that shows • no. of different value taken by Random Variable and • their corresponding probabilities Sum of all probabilities of distribution will always be equal to 1
Expected Value	It is also mean of probability distribution. $\mu = E(X) = \sum PX$
Variance	$\sigma^2 = E(X^2) - [E(X)]^2$ $\sigma^2 = \sum PX^2 - [\sum PX]^2$

PYQ May 18

Variance of a random variable x is given by

a. $E(X-\mu)^2$ b. $E[X-E(X)]^2$ c. $E(X^2-\mu)$ d. a or b

Ans: d

PYQ Nov 19

What is the probability of occurring 4 or more than 4 accidents.

No.	0	1	2	3	4	5	6	7
Freq.	36	27	33	29	24	27	18	9

a. 24 b. 69 c. 78 d. 80

Ans: c

PYQ Jul 21

The value of k for the below probability distribution of X is

X	0	1	2	3	4	5	6
P(X)	5k	3k	4k	6k	7k	9k	11k

a. 39 b. 1/40 c. 1/49 d. 1/45

Ans: d

PYQ Dec 21

Assume that a probability for rain in a day is 0.4. An umbrella salesman can earn ₹400 per day in case of rain on that day and will lose ₹100 per day if there is no rain. The expected earnings in ₹ per day of the salesman is

a. 400 b. 200 c. 100 d. 0

Ans: c

PYQ Dec 21

Find SD for probability distribution given below:

X	1	2	4	5	6
P	0.15	0.25	0.2	0.3	0.1

a. 1.49 b. 1.56 c. 1.69 d. 1.72

Ans: c