MATHS MAGIC

MATHS HANDBOOK

By

MATHS MAGICIAN

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SWAPNIL PATNI CLASSES

RATIO & PROPORTION

OPERATION ON RATIO :	OPERATION ON PROPORTION :
• Inverse ratio → $\frac{a}{b} \Rightarrow \frac{b}{a}$	• Invertendo→ $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c}$
• Duplicate ratio → $\frac{a}{b} \Rightarrow \frac{a^2}{b^2}$	• Alternendo→ $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{c} = \frac{b}{d}$
• Sub-duplicate ratio → $\frac{a}{b} \Rightarrow \frac{\sqrt{a}}{\sqrt{b}}$	• Componendo→ $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{b} = \frac{c+d}{d}$
• Triplicate ratio → $\frac{a}{b} \Rightarrow \frac{a^3}{b^3}$	• Dividendo→ $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a-b}{b} = \frac{c-d}{d}$
• Sub-triplicate ratio → $\frac{a}{b} \Rightarrow \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$	• Componendo-dividendo→ $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$
• Compounded ratio → $\frac{a}{b} \times \frac{c}{d} \times \frac{e}{f}$	

INDICES & LOGARITHMS

INDICES	LOGARITHMS
$a^{m/n} = \sqrt[n]{a^m}$ Here, a = base m = power n = root	Conversion of log into indices $\log_a m = n$ then $a^n = m$
• $a^m \times a^n = a^{m+n}$ • $\frac{a^m}{a^n} = a^{m-n}$ • $(a^m)^n = a^{mn}$ • $a^0 = 1$ • $a^{-m} = 1/a^m$ • $a^m = 1 / a^{-m}$	• $\log_a m + \log_a n = \log_a mn$ • $\log_a m - \log_a n = \log_a (m/n)$ • $\log_a (m^n) = n \log_a m$ • $\log_a 1 = 0$ • $\log_a a = 1$ • $\log_a b = \frac{\log_c b}{\log_c a}$

❖ **Do you know this**→$(a + b)^3 = a^3 + b^3 + 3ab\,(a + b)$

- $(a - b)^3 = a^3 - b^3 - 3ab\,(a - b)$
- $a^3 + b^3 = (a + b)\,(a^2 - ab + b^2)$
- $a^3 - b^3 = (a - b)\,(a^2 + ab + b^2)$
- $a^2 - b^2 = (a + b)\,(a - b)$

EQUATIONS

- **Quadratic Equation →** $ax^2 + bx + c = 0$ $\qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- $\alpha + \beta = -b/a$ $\qquad \alpha.\beta = c/a$
- $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $\qquad (\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$
- $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$ $\qquad \alpha^3 - \beta^3 = (\alpha - \beta)(\alpha^2 + \beta^2 + \alpha\beta)$

TIME VALUE OF MONEY

- **<u>Simple interest :</u>**

  - Interest is paid only once at the end of time

  $I = (Pnr)/100$ $\qquad A = P + I$ $\qquad A = P\left[1 + \frac{nr}{100}\right]$

Here, P =principle = initial money deposited R = rate of interest

N = number of year = number of month /12= number of days / 365

- **<u>Compound interest :</u>**

  $A = P(1 + i)^n$ $\qquad I = A - P$

Here,N = number of conversion period = no of years * (1or or 4 or 12)

I =(rate of interest) / (1 or 2 or 4 or 12)

Note : When n =1 & interest is paid annually then Simple interest = compound interest

- **<u>Applications of compound interest :</u>**

a. In the problems of population :

$A = P(1 + i)^n$

here, A = final population p = initial population

i = rate of growth of population = birth rate – death rate

b. In the problems of depreciation :

$SV = C P (1 - i)^n$

SV = scrap value CP = cost price I = rate of depreciation n = effective life

c. Effective rate of interest :

$i_e = (1 + i)^n - 1$

Where,ie= effective interest rate i = actual / nominal interest rate n = 1 year * 1/ 2 / 4/ 12

- **Future value:**

a. By annuity regular : (payment at end)

$F . V = \frac{A\left[(1+i)^n - 1\right]}{i}$ where, A = annuity

b. By annuity due : (payment at start)

$F . V = \frac{A\left[(1+i)^n - 1\right]}{i} \times (1 + i)$

- **If installments are paid initially & total amount is to be received after certain years then use future value formula.**
- **Future value is also used for sinking fund problems.**

- **Present value :**

a. By annuity regular :

$V = \frac{A\left[(1+i)^n - 1\right]}{i(1+i)^n} = A . P(n, i)$

- **If total amount is received initially & installments are paid later on then use present value.**
- **Present value is applicable in the problems of house property, loan or borrow.**
- **Amount of loan, amount of money borrowed& amount of house property is taken as present value.**

SET, FUNCTION & RELATION

SET :-Set is group of things. It is represented by { }.

Null set →It is a set containing 0 no. of elements. It is given by ϕ or { }.

For equal sets → Set A = Set B

For equivalent sets →n(A) = n(B)

SUBSET : Set B is said to be subset of set A if all the elements of set B belong to set A.

No. of subsets → 2^n

Number of proper subsets → $2^n - 1$ Number of improper subset → 1

THEOREM OF ADDITION→For two sets A & B- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

For 3 sets A, B & C: $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

DERIVATIVE

- $\frac{d}{dx}(x^n) = n \cdot x^{n-1}$ $\frac{d}{dx}(e^x) = e^x$

  $\frac{d}{dx}(\log x) = 1/x$ $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

  $\frac{d}{dx}(a^x) = a^x \cdot \log_e a$ $\frac{d}{dx}(x) = 1$

  $\frac{d}{dx}(k) = 0$ $\frac{d}{dx} \cdot \frac{1}{x} = -\frac{1}{x^2}$

- $y = u \pm v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$
- $y = u \cdot v \Rightarrow \frac{dy}{dx} = V \cdot \frac{du}{dx} + u \cdot \frac{dv}{dx}$
- $y = u / v \Rightarrow \frac{dy}{dx} = \frac{V \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{V^2}$
- $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ when $y = f(t)$ & $x = g(t)$
- $y = f(x)^{g(x)}$ then $\log y = \log f(x)^{g(x)}$

CORRELATION & REGRESSION

- **Bivariate data** : data made up of **2 variable** at **same** point of **time**. For m × n distribution: No. of **marginal distribution : 2** __ No. of **conditional** distribution : **m + n** ___ methods of analysis : correlation & regression.
- **Correlation** : cause & effect relationship between two variable__ **states extent & value of relation**___ can't give mathematical relation or formula between 2 variable
  Regression : gives mathematical relation__ **gives value of dependent variable from independent variable**
- **Correlation** : +ve or -ve.____ **-1≤ r ≤1** ___ coeff. of correlation r has **no unit** (it is relative measure) __ not affected by both change of scale & origin___ positive relation : one increases & other increases e.g. height & wt of person, income & expense, speed of car & distance covered after applying brakes, rainfall & crop production ___ Negative relation : one increases & other decreases e.g. price & demand, day temp & sale of woolen clothes__ No relation : e.g. size of shoes & intelligence___ Methods of correlation (4) :
  a. **Scatter diagram** : r is +ve : points from lower left to upper right __ r is -ve : points from upper left to lower right___ If all points are on a line then perfect +ve (agreement r = +1) or perfect -ve (disagreement r = -1) relation. __ scatter diagram may be linear or curvilinear__ gives only sign of relation but not its extent.
  b. **Karl pearson's** product moment correlation :**used** only when data is **quantitative, relation is linear, variation is less.** $r = \frac{cov(x, y)}{\sigma_x . \sigma_y}$
  c. **Spearman's rank correlation method** : used when data is **qualitative**, relation is linear or non-linear__ can't be used for bivariate data__ if $\Sigma d^2 = 0$ then r=1 $r = 1 - \left[\frac{6\sum d^2}{n(n^2 - 1)}\right]$
  d. **Concurrent deviation method** : used **when magnitude of data is not much important** __ **quickest** method of correlation $r = \pm \sqrt{\frac{\pm (2c - m)}{m}}$
- **Probable error** : difference between r of sample & r of population **P.E(r) =** $\frac{0.6745\ (1-r^2)}{\sqrt{n}}$
  If r < (P.E.) then no significant relation ___ if r > 6 .(PE) then significant relation
- Coefficient of determination = ratio of explained variance to total variance = r^2
  coeff. of non-determination = $1-r^2$
- Effect of scale & origin on r → If u = a + bx& v = c + dx
  b = - x / u & d = -y / v
  If x changes into x or y into y then change of scale is

b	**+**	**-**	**+**
d	**+**	**-**	**-**
r_{uv} =	r_{xy}	r_{xy}	- r_{xy}

  1.
- **Regression** : uses **least square principle** __ 2 types of line : **x on y (used when y is given & x is unknown)**&**y on x (used when x is given & y is unknown)**

  Y on X → $y = a + b.x \rightarrow (y-\bar{y}) = b_{yx} (x -\bar{x}) \rightarrow b_{yx} = r \times \frac{\sigma_y}{\sigma_x} = \frac{-coeff.ofx}{coeff.ofy}$

X on Y → x = a + b.y→ $(x-\bar{x}) = b_{xy}(y-\bar{y})$ → $b_{xy} = r \times \frac{\sigma_x}{\sigma_y} = \frac{-coeff.ofy}{coeff.ofx}$

❖ **$r = \pm\sqrt{byx \cdot bxy}$** -1 ≤ r ≤ 1 $|byx.bxy| \leq 1$

PROBABILITY & EXPECTED VALUE

❖ 0≤ P(A) ≤1 **P(A) = 0 … impossible event P(A) =1 …. Sure event**

❖ Simple event : which can't be split into 2 parts e.g . getting a head
Compound/complex event : which can be split into two or more parts e.g. tossing of a coin (2 parts- head & tail)

❖ Mutually **exclusive**event : can't occur simultaneously **P (A∩B) = 0**
Exhaustiveevents : any one of them will surely occur**P(A∪B) = 1**
Equally likelyevents : probability are equal **P(A) = P(B)** but event may be same or different
If A & B are exclusive, exhaustive , equally likely then P(A) =P(B)= 1/2
If A & B & C are exclusive, exhaustive , equally likely then P(A) =P(B)= P(C)=1/3

❖ Two methods:a. Subjective probability : Dependent on personal judgement b. Objective probability

❖ **Compound probability or joint probability:**The probability of occurrence of two events A and B simultaneously is known as the Compound Probability or Joint Probability of the events A and B and is denoted by P(A∩B).
Two types of compound probability :
a) **Dependent events** :**P(A/B) = P (A∩B) / P(A) orP (A∩B) = P(A/B) . P(A)**
b) **Independent events** :**P(A /B) = P(A)**&**P(A ∩B) = P(A) .P(B)**
If A & B are independent then A, B' & A', B & A', B'are also independent

❖ **Expected value:- (mean)** $E(x) = \sum xP$ $E(x^2) = \sum x^2 P$
Variance = $V(x) = E(x^2) - [E(x)]^2$

Properties of expected value: Affected by both change of scale & change of origin : If Y = a+ b.Xthen **E(Y) = a + b. E(x)___ E(xy) = E(x) x E(y)___E(k x) = k.E(x)___E(x + y) = E(x) + E(y) ____E(k) = k** for any constant k

❖ Odds in favour of an event = $\frac{number\ of\ ways\ favourable\ to\ event}{number\ of\ non\text{-}\ favourable\ ways} = \frac{p}{q}$

Odds against an event = $\frac{number\ of\ ways\ non\text{-}\ favourable\ to\ event}{number\ of\ favourable\ ways} = \frac{q}{p}$

Probability = $\frac{p}{p+q}$

❖ **Theorem of addition** : P(AUB) = P(A) + P(B) – P(A ^B)

STATISTICAL DESCRIPTION OF DATA

- **Origin of word statistics** : Latin : status __ Italian : statista__German : statistik __French : statistique

- **Definition of Statistics: a.** As a **plural noun**: defined as **data qualitative as well as quantitative**, that are collected, usually with a view of having statistical analysis.**b.** As a **singular noun**: defined, as the scientific method that is employed **for collecting, analysing and presenting data.**
- **Limitations of Statistics :** deals with the **aggregates,** not with individual___**concerned with quantitative** data.

- **Collection of data :**
  - **a. Interview method :** Personal Interview method (best for natural calamity like cyclone, earthquake , epidemic like plague)__Indirect Interview (best for rail accident) ___Telephone interview (quickest and non-expensive)
  - **b. Mailed questionnaire method** : (covers widest area)
  - **c. Observation method** : (time consuming, laborious and covers only a small area.)

- **Scrutiny of data :** To detect error__ Used for internal consistency __Applicable if there may be two or more series of figures which are in some way or other related to each other e.g density = population / area

- **Types of data :**
  - a. **Quantitative data:**termed as variable. Discrete data- It has fixed value. Discrete data with frequency is known as ungrouped frequency data. __Continuous data- known as grouped frequency data. E.g. Height, weight, profit, loss etc.
  - b. **Qualitative data:**can not be measured by numerical value. It includes characteristics or qualities. This is known as attribute.e.g. colour of a person, intelligence, nationality, gender.
  - c. **Time-series or chronological data:**This varies according to time.
  - d. **Geographical data:**It varies with space.

- **Types of data according to method of collection:-*Primary data :*** It is data collected personally by a person or agency. ___ ***Secondary data :*** A data which uses primary data as basis is called as secondary data. Sources of secondary data are- national & international organizations, ministry of different departments etc

- **Methods of presentation of data:-**Textual method___Tabular method: Data is presented in the table. It contains rows & columns___Diagramatic method: Data is expressed by diagrams. It is most attractive method.**Types of diagrams:**
  - **a. Line diagram:**used for time-series data. Multiple line diagrams- for comparing two data with same unit. Multiple axis diagram- for comparism of data with different unit. In ratio chart, data is presented in the form of logarithms.
  - **b. Bar diagrams:**Vertical bar diagram for quantitative data & horizontal for qualitative data. For comparing two or more data, multiple or grouped bar diagrams are used.
  - **c. Pie chart:** For showing a total data in smaller groups, pie chart is used which is in the form of percentage or angle.

- **Types of frequency distribution diagram:**

a. **Histogram:** is in the form of vertical bar & used for continuous data__ gives information about mode.
b. **Frequency polygon/ frequency curve :** smooth curve for which the total area is taken to be unity ___freq. curve is limiting form of a histogram or frequency ___four types of frequency curve : Bell-shaped curve : most commonly used shape e.g. distribution of height, weight, mark, profit etc.___U-shaped curve ___ J-shaped curve __Mixed curve.
c. **Ogive:** It is graph of less than or more than cumulative frequency against given data. It gives median, quartiles, decile, percentile.

- **Inclusive data** includes both limits. E.g. 0-9, 10-19, 20-29 etc**.Exclusive** data excludes upper limit of each class. Here, class limits & boundaries are same.e.g. 0-10, 10-20, 20-30 etc.
- **Relative frequency**:It is ratio frequency of given class to the total frequency.R.F. = $\frac{f}{N}\times 100$
- Frequency density:It is ratio of frequency of given class to its width.F.D. = $\frac{f}{h}$

CENTRAL TENDENCY

Central tendency may be defined as the tendency of a given set of observations to cluster around a single central or middle value and the single value that represents the given set of observations is described as a measure of central tendency or, location or average.

	A.M.	Median	Mode	G.M.	H.M.
Best / most commonly used	Yes				
Most popular			Yes		
Based on all observations	Yes			YES	Yes
Have mathematical property	Yes			Yes	Yes
Affected by sampling fluctuations	Yes very much	No			
Easy to calculate			Most easy &fast	Most difficult	
For open end class		can be used (best)	Can be used		
			Sometimes can't be defined		

- Mean- best measure of central tendency, have mathematical property, used for finding average speed when time is constant
  Median- best for open end class, not affected by extreme value
  Mode- can have multiple values, can't be defined every time
  G.M – difficult to compute , used for finding average in case of rate, interest, percentage
  H.M. - used for finding average speed when distance is constant

- Sum of deviation about mean – zero ___Sum of squares of deviation about mean – minimum_____Sum of absolute deviation about median - minimum
- A.M, median, mode – affected by both change of scale (multiplication/ division) & change of origin (addition/ subtraction)
  { if 3x + 4y =8 then → $3.\bar{x} + 4.\bar{y}$ =8 →$3.Me_x + 4.Me_y = 8$ → $3.Mo_x + 4.Mo_y = 8$ }
- Range, mean deviation , standard deviation, quartile deviation – affected only by change of scale (multiplication/ division) & not by change of origin (addition/ subtraction)
  { if y = a + b.x then → $R_y = |b| R_x$→ $M.D_y = |b| M.D._x$→ $\sigma_y = |b| \sigma_x$→ $Q.D._y = |b| Q.D._x$ }
- For 2 numbers a & b, A.M. = (a + b) /2 G.M.= $\sqrt{ab}$ H.M. = $\frac{2ab}{a+b}$ ∴$(G.M)^2$ = A.M. * H.M.
- A.M ≥ G.M. ≥ H.M.
- Variance = σ^2coeff. of variation (C.V)= $\frac{\sigma}{\bar{x}}$ * 100 { less C.V. →more consistency →more stability}
- Combined std. deviation σ_{12} = $\sqrt{\frac{[N_1 (\sigma_1^2 + d_1^2) + N_2 (\sigma_2^2 + d_2^2)]}{N1+N2}}$ Combined mean X_{12}= $\frac{N1\,\overline{X_1} + N2\,\overline{X_2}}{N1+N2}$

THEOROTICAL DISTRIBUTION

- **Binomial distribution** → biparametric (n,p) → P = F(x) = ^nc_x . p^x .q^{n-x}

Mean= np variance = npq max. variance = n /4 mode (unimodal or bimodal) – (n+1)p

Mean is always more than variance.

- **Poisson's distribution** → uniparametric (m= np) → $P = F(x) = \frac{e^{-m} \cdot m^x}{x!}$

  Mean= m variance = m mode (unimodal or bimodal) – m
- **Normal distribution** → **Symmetric curve** Mean = Mode = Median = μ (It is unimodal.)

  Variance = σ^2

  Mean deviation = 0.8σ Q.D. = $0.675\ \sigma$

INDEX NUMBER

- Value= price × quantity Index no. of base year is 100
- **Simple aggregative method** - $P_{on} = \frac{\sum P_n}{\sum P_o} \times 100$ **Simple relative method** - $P_{on} = \frac{\sum P_n / P_o}{N} * 100$
- **Weighted relative method** - $P_{on} = \frac{\sum \frac{Pn}{Po}.w}{W} * 100$
- **Laspayres** method – weightage (**base** yr.) – $\frac{\sum PnQo}{\sum PoQo} * 100$
- **Paasches** method – weightage (**current** yr.) – $\frac{\sum PnQn}{\sum PoQn} * 100$
- **Fisher's method** – $\sqrt{Laspayres * paasches}$
- Test of adequacy :
  - **Unit test**→ satisfied by **all method except simple aggregative** method.
  - **Factor reversal** test → $P_{on} * Q_{on} = V_{on}$ → satisfied by **fishers** test only.
  - **Time reversal** test → $P_{10} \times P_{01} = 1$ → satisfied by **fishers** test only.
  - **Circular** test → test of shifting of base→ extension of time reversal test →Satisfied only by **weighted aggregative, simple aggregative & geometric mean method**

SEQUENCE AND SERIES

- **Arithmetic progression :** $t_n = a + (n-1)\,.\,d$ $s_n = n/2\,[2a + (n-1)\,d]$ $s_n = n/2\,[t_1 + t_n]$
- **Geometric progression :** $t_n = a\,.\,r^{n-1}$ $S_n = a\,\frac{(r^n - 1)}{r - 1}$ $S_\infty = \frac{a}{1-r}$
- Sum of n natural numbers $\sum n = \frac{n(n+1)}{2}$ Sum of squares of n natural numbers $\sum n^2 = \frac{n(n+1)(2n+1)}{6}$ Sum of cubes of n natural numbers is $\sum n^3 = \left[\frac{n(n+1)}{2}\right]^2$

CHAPTER 1

INDICES & LOGARITHMS

1. The value of $\left[\frac{2p^2q^3}{3xy}\right]^0$ is equal to

a. 0 b.2/3 **c. 1** d. None

2. $X^{(a-b)} * X^{(b-c)} * X^{(c-a)}$ =

a. **1** b. 0 c. 3 d. x

Solution : $X^{a-b+b-c+c-a} = X^{(a-b)} = X^0 = 1$

3. $2 * (8)^{1/3}$ =

a. **4** b. 1 c. 8 d. none

Solution : 2 * 2 = 4

4. $\left[\frac{81x^4}{y^{-8}}\right]^{\frac{1}{4}}$ has simplified value equal to

a. xy^2 b. x^2y c. $9xy^2$ **d. None**

Solution:

$\left[\frac{81x^4}{4^{-8}}\right]^{1/4} = \left[\frac{81x^4}{\frac{1}{y^8}}\right]^{1/4} = [81x^4y^8]^{1/4} = [81]^{\frac{1}{4}} \times x^{4\times\frac{1}{4}} y^{8\times\frac{1}{4}} = 3 \times x^1 \times y^2 = 3xy^2$

5. $(32 / 243)^{-1/5}$

a. 2/3 **b. 3/2** c. 0 d. none

Solution:

$\left[\frac{32}{243}\right]^{\frac{-1}{5}} = \frac{\frac{1}{32^{\frac{1}{5}}}}{\frac{1}{243^{\frac{1}{5}}}}$ $= \frac{1}{2} \times \frac{3}{1} = \frac{3}{2}$

6. $(x^{b+c})^{b-c}$ $(x^{c+a})^{c-a}$ $(x^{a+b})^{a-b}$ is equal to

a. 0 **b 1** c x d $\frac{1}{x}$

Solution:

$(x^{b+c})^{b-c} \times (x^{c+a})^{c-a} \times (x^{a+b})^{a-b} = x^{b^2-c^2} \times x^{c^2-a^2} \times x^{a^2-b^2} = x^{b^2-c^2+c^2-a^2+a^2-b^2} = x^0 = 1$

7. $\left[\frac{x^l}{x^m}\right]^{l^2+lm+m^2} \times \left[\frac{x^m}{x^n}\right]^{m^2+mn+n^2} \times \left[\frac{x^n}{x^l}\right]^{l^2+ln+l^2}$

a. 0 b. x **c.1** d. none

8. Value of $(a^{1/8} + a^{-1/8})(a^{1/8} - a^{-1/8})(a^{1/4} + a^{-1/4})(a^{1/2} + a^{-1/2})$ is

a. $a + \frac{1}{a}$ b. $a - \frac{1}{a}$ c. $a^2 + \frac{1}{a^2}$ d. $a^2 - \frac{1}{a^2}$

Solution:

$$\left(a^{1/8} + a^{-1/8}\right)\left(a^{1/8} - a^{-1/8}\right)\left(a^{1/4} + a^{-1/4}\right)\left(a^{\frac{1}{2}} + a^{-\frac{1}{2}}\right)$$

$$[a^{1/8}]^2 - [a^{-1/8}]^2 \left[a^{1/4} - a^{-1/4}\right] \left[a^{1/4} + a^{-1/4}\right] \left[a^{1/2} + a^{-1/2}\right]$$

$$\left[a^{1/2}\right]^2 - \left[a^{-\frac{1}{2}}\right] \left[a^{1/2} + a^{-1/2}\right] = a^1 - a^{-1} = a - \frac{1}{a^1}$$

CHAPTER 2

STATISTICAL DATA

1. Initially, statistics was mostly related with
- a) **State** c) Economics
- b) Accounts d) None.

2. Word 'statistics' is defined in _____ sense
- a) 1 c) 3
- b) **2** d) None

3. In singular sense statistics is defined as :
- a) Data quantitative & Qhalitative c) Both a) and b)
- b) **Scientific method of collection, analysis & presentation** d) None.

4. Which of the following is best method of collection of data –
- a) Interview method c) Observation,
- b) **Mailed questionaire** d) None.

5. Chronological or temporal data is another name of –
- a) Geographical data c) Attribute
- b) **Time series data** d) None.

6. Line diagram is mostly drawn for –
- a) Geographical data c) **Time series data**
- b) Attribute d) None.

7. Which of the method is useful for educated & uneducated people both –
- a) Tabular **c) Diagramatic**
- b) Tentual d) None

8. When time series data has large variations the which of the following diagram is used –
- a) Bar diagram c) Pie Chart
- b) **Ratio chart** d) None

9. For logarithmic data, which of the following diagram is used –
- a) Bar digram c) Pie chart
- b) **Ratio chart** d) None

10. Horizontal bar diagram is used for –
- a) Time series data c) Qualitative data
- b) Spatial data **d) Both b & c**

11. Vertical bar diagram is drawn for :
- a) Time series data c) Spatial data
- b) Quantitative data **d) both a & b**

12. There are _____ types of Frequency distribution –
- a) 1 c) 3
- b) **2** d) None.

13. Tabulation of discrete random variable is known as
- a) Discrete frequency distribution
- b) Ungrouped frequency distribution,
- c) Simple frequency distribution,
- **d) All of these.**

14. Grouped frequency distribution is related with –

a) Discrete variable c) Both a & b

b) **Continuous Variable** d) None

15. Cumulative frequency only refers to –

a) **Less than C.F.** c) Both a & b

b) More than C.F. d) None

16. Ration of class frequency to total frequency is –

a) **Relative frequency** c) Percentage frequency

b) Frequency density d) None.

17. Ratio of class frequency to total frequency, expressed as a percentage is called as –

a) Relative frequency **c) Percentage frequency**

b) Frequency density d) None

18. Sum of all relative frequency is –

a) 0 c) 100

b) **1** d) None.

19. Sum of all percentage frequency is :

a) 0 **c) 100**

b) 1 d) None

20. Area diagram is another name of –

a) **Histogram** c) Ogive

b) Frequency poly gon d) None

21. We obtain, _____ from histogram,

a) AM **c) Mode**

b) Median d) None

22. Frequency polygon is suitable for –

a) Simple frequency distribution

b) Grouped frequency distribution,

c) Both a & b

d) None.

23. Cumulative frequency diagram is another name of –

a) Histogram **c) Ogive**

b) Frequency polygon d) None

24. Ogive is of _____ types –

a) 1 c) 3

b) **2** d) None

25. Frequency curve is limiting form of –

a) Histogram **c) a) or b).**

b) Frequency polygon d) None

26. The data obtained from a newspaper are

(a) Primary data **(b) Secondary Data** (c) Both (a) and (b) (d) None of these

27. In an exclusive type distribution, the limits excluded are

(a) **Upper limits** (b) Lower limits

(c) either of the lower or upper limits (d) lower limits and upper limits both

28. The heading of the rows given in the first column of a table are called
(a) **Stubs** (b) Captions (c) Sub titles (d) Prefatory notes

29. The column heading of a table are known as
(a) Sub-titles (b) Stubs (c) Reference notes **(d) Captions**

30. The median of a given frequency distribution is found graphically with the help of
(a) Pictogram (b) Pie Chart (c) Frequency curve **(d) Ogive**

31. The amount of non-responses is maximum in _____.
(a) **Mailed questionnaire method** (b) Interview method (c) Observation method (d) All of these

32. The quickest method to collect primary data is _____.
(a) Personal interview (b) Indirect interview **(c) Telephone interview** (d) By observation

CHAPTER 3

PART A : CENTRAL TENDENCY

1. The mean for a symmetrical distribution is 50.6. Find the values of median and mode.
i. 56 ii. 65 iii. 50.6 iv. none

$$Solution: \ \bar{x} = Median = Mode = 50.6$$

2. In a moderately asymmetrical distribution –The mode and median are 300 and 240 respectively. Find the value of mean.
i. 210 ii. 240 iii. 350 iv. None

Solution :
$\bar{x} - Mode = 3\,(\bar{x} - Median)$

$\bar{x} - 300 = 3\,(\bar{x} - 240)$ $\quad \therefore \bar{x} = 210$

3. If there are two groups containing 30 and 20 observations and having 50 and 60 as arithmetic means, then the combined arithmetic mean is :
i) 55 ii) 56 iii) 54 iv) 52.

Solution :
$N1 = 30 \quad N2 = 20 \quad \bar{x}1 = 50 \quad \bar{x}2 = 60$

$\bar{x}12 = \frac{N1\bar{x}1 + N2\,\bar{x}\,2}{N1+N2} = \frac{30\times50+20\times60}{30+20} = 54$

4. The average salary of a group of unskilled workers is Rs. 10000 and that of a group of skilled workers is Rs. 15,000. If the combined salary is Rs. 12000, then what is the percentage of skilled workers?
i) 40% ii) 50% iii) 60%iv) none of these.

Solution : $N1 = x \quad N2 = 100 - x \quad \bar{x}1 = 15000 \quad \bar{x}2 = 10000$

$\bar{x}12 = 12000 \quad N1 + N2 = 100$

$\bar{x}12 = \dfrac{N1\bar{x}1 + N2\bar{x}2}{N1 + N2}$

$12000 = \frac{x \times 15000 + (100-n) \times 10000}{100}$ $\quad\quad\quad\quad$ $\therefore x = 40$

5. The average rainfall to a week excluding Sunday was 10 cms. Due to heavy rainfall on Sunday, the average rainfall for the week rose to 15 cms. How much rainfall was there on Sunday?

a) 55 **b. 45** c. 40 d. none

Solution : $N1 = 6 \quad N2 = 1 \quad N1 + N2 = 7 \quad\quad \bar{x}1 = 10 \quad \bar{x}2 = \ ? \quad \bar{x}12 = 15$

6. If there are two groups with 75 and 65 as harmonic means and containing 15 and 13 observation then the combined HM is given by

i) 65 ii) 70.36 iii) 70 iv) 71.

Solution : $N1 = 15 \quad N2 = 13 \quad\quad H1 = 75 \quad H2 = 65$

$$H12 = \frac{N1+N2}{\frac{N1}{H1}+\frac{N2}{H2}} = \frac{15+13}{\frac{15}{75}+\frac{13}{65}} = 70$$

7. If a constant 25 is added to each observation of a set, the mean of the set is

(A) increased by 25 (b) decreased by 25
(c) 25 times the original mean (d) zero

8. Two variables x and y are given by y = 2x – 3. If the median of x is 20, what is the median of y?

i) 20 ii) 40 iii) 37 iv) 35.

Solution : $y = 2x - 3$

$Median\ of\ y = 2\ Median\ of\ x - 3$

$= 2 \times 20 - 3 \ = 37$

9. Mean of two numbers is 16 & their geometric mean is 8. What is harmonic mean?

a. 8 b. 24 c. 4 d. 128

Solution : $GM^2 = AM \times HM$

$(8)^2 = 16 \times HM \quad\quad 64 = 16 \times HM \quad\quad 4 = HM$

10. A cyclist pedals from his house to college at a speed of 10 km. per hour and back from the college to his house at 15 km. per hour. Compute his average speed.

a) 10 b) 12 c) 20 d) none

Solution : $S1 = 10 \quad S2 = 15$

Distance is same, so use HM Avg.Speed = HM = $\frac{2ab}{a+b} = \frac{2\times10\times15}{10+15} = \frac{300}{25} = 12$

11. An aeroplane flies from A to B at the rate of 500 km/hour and comes back from B to A as the rate of 700 km/hour. The average speed of the aeroplane is :

i) 600 km. per hour iii) 100 $\sqrt{35}$ km. per hour.

ii) 583.33 km. per hour. iv) 620 km. per hour.

Solution : $S1 = 500 \quad S2 = 700$

Avg.Speed = HM = $\frac{2ab}{a+b} = \frac{2\times500\times700}{500+700} = 583.33$

12. The average age of 15 students of a class is 15 years. Out of them, the average age of 5 students is 14 years and that of the other 9 students is 16 years. The age of the 15th student is:

(a) 11 years
(b) 14 years
(c) 15 years
(d) None

Solution : $N1 = 5 \quad N2 = 9 \quad N3 = 1 \qquad N1 + N2 + N3 = 15$

$\bar{x}1 = 14 \qquad \bar{x}2 = 16 \qquad \bar{x}3 = \qquad \bar{x}123 = 15$

$$\bar{x}123 = \frac{N1\bar{x}1 + N2\bar{x}2 + N3\bar{x}3}{N1 + N2 + N3}$$

$15 = \frac{S\times14+9\times16+1\times\bar{x}3}{S+9+1} \qquad \bar{x}3 = 11$

13. For open-end classification, which of the following is the best measure of central tendency?

i) AM ii) GM **iii) Median** iv) Mode

14. The presence of extreme observations does not affect :

i) AM **ii) Median** iii) Mode iv) Any of these.

15. Which one of the following is not uniquely defined ?

i) Mean ii) Median **iii) Mode** iv) All of these

16. The algebraic sum of deviations of observations from their A.M. is

a) 2 b) -1 c) 1 **d) 0.**

17. G.M. of a set of n observations is the _____ root of their product.

a) n/2th b) (n+1) th **c) nth** d) (n-1) th.

18. G.M. is less than H.M.

a) True **b) false** c) both d) none.

19. The value of the middlemost item when they are arranged in order of magnitude is called.

a) Standard deviation b) Mean c) Mode **d) Median.**

20. The value which occurs with the maximum frequency is called.

a) Median **b) mode** c) mean d) none.

21. Which measure(s) of central tendency is (are) considered for finding the average rates ?

i) AM ii) GM iii) HM **iv) Both (ii) and (iii).**

22. Which of the following results hold for a set of distinct positive observations ?

i) AM ≤ GM ≤ HM iii) **AM > GM > HM**

ii) HM ≤ GM ≤ AM iv) GM > AM > HM

23. When a firm registers both profits and losses, which of the following measure of central tendency cannot be considered ?

i) AM **ii) GM** iii) Median iv) Mode.

24. Quartiles are the values dividing a given set of observations into :

i) Two equal parts

ii) Four equal parts

iii) Five equal parts

iv) None of these.

25. Quartiles can be determined graphically using :

i) Histogram **iii) Ogive**

ii) Frequency Polygon iv) Pie Chart.

CHAPTER 4

DISPERSION

1. The range of 15, 12, 10, 9, 17, 20 is

a) 5 b) 12 c) 13 d) 11.

Solution : $Range = L - S = 20 - 9 = 11$

2. Range for following data is,

X	2	4	6	8	10
F	7	9	5	4	3

a)4 b)8 c) 12 d) none

Solution : $Range = L - S = 10 - 2 = 8$

3. The mean and S.D. of 1, 2, 3, 4, 5, 6 is

a) 7/2, √35/12 b) 7/2, √3

c) 3, 3 d) 3, 35/12

solution :

X	$d = x - \bar{x}$	d^2
1	-2.5	6.25
2	-1.5	2.25
3	-0.5	0.25
4	0.5	0.25
5	1.5	2.25
6	2.5	6.25
		17.5

$\bar{x} = \frac{\sum x}{N} = \frac{21}{6} = 3.5$ $\sigma = \sqrt{\frac{\sum d^2}{N}} = \sqrt{\frac{17.5}{6}} = \sqrt{\frac{35}{12}}$

4. The coefficient of variation of a series is 58. Its S.D is 21.2. Its arithmetic mean is

a) 36.6 b) 22.6

c) 26.6 d) 36.1

Solution :

$C.V = \frac{\sigma}{\bar{x}} \times 100$ $58 = \frac{21.2}{\bar{x}} \times 100$ $\bar{x} = 36.6$

5. Which of the following companies A and B is more consistent so far as the payment of dividend is concerned?

Dividend paid by A :	5	9	6	12	15	10	8	10
Dividend paid by B :	4	8	7	15	18	9	6	6

a) A b) B

c) Both a) and b) d) Neither a) nor b)

solution :

$Find\ C.V_A = \frac{\sigma}{\bar{x}} \times 100$ $C.V_B = \frac{\sigma}{\bar{x}} \times 100$

$C.V_A < C.V_B$

$\therefore A\ is\ more\ Consistent$

6. If all the observations are multiplied by 2, then

a) New SD would be also multiplied by 2

b) New SD would be half of the previous SD

c) New SD would be increased by 2

d) New SD would be decreased by 2.

7. If X and Y are related as 3x - 4y = 20 and the quartile deviation of x is 12 then, then the quartile deviation of y is:

(a) 14

(b) 15

(c) 16

Solution :$Q.Dy = |b| \times Q.Dx = \left|\frac{-x}{y}\right| \times QDx = \left|\frac{-3}{4}\right| \times 12 = 9$

8. If two variables x and y are related by 2x+3y-7=0 and the mean and mean deviation about mean of x are 1 and 0.3 respectively. Then the coefficient of mean deviation of y about mean is :

a) -5 b) 12

b) 50 c) 4

solution : $\bar{x} = 1$ $2x + 3y - 7 = 0$ $2\bar{x} + 3\bar{y} - 7 = 0$

$Put \bar{x} = 1$ then $\bar{y} = \frac{5}{3}$

$M.Dx = 0.3$

$MDy = |b| \times MDx = \left|\frac{-x}{y}\right| \times MDn = \left|\frac{-2}{3}\right| \times 0.3 = 0.2$

Coeff.of Mdy= $\frac{MDy}{\bar{y}} \times 100 = 12$

9. If two samples of sizes 30 and 20 have means as 55 and 60 and variances as 16 and 25 respectively, then what would be the S.D of the combined sample size 50?

a) 5.00 b) 5.06

c) 5.23 d) 5.35

solution :

$N1 = 30$ $N2 = 20$ $\bar{x}1 = 55$ $\bar{x}2 = 60$ $\sigma 1 = 4$ $\sigma 2 = 5$

$\bar{x}12 = \frac{N1\bar{x}1 + N2\bar{x}2}{N1+N2} = \frac{30\times55+20\times60}{30+20} = 57$

$d1 = \bar{x}12 - \bar{x}1 = 57 - 55 = 2$ $d2 = \bar{x}12 - \bar{x}2 = 57 - 60 = -3$

$$\sigma_{12} = \sqrt{\frac{N1(\sigma1^2 + d1^2) + N2(\sigma2^2 + d2^2)}{N1 + N2}}$$

= 5.06

17. When it comes to comparing two or more distribution, we consider

a) **Relative measures of dispersion**

b) Absolute measures of dispersion

c) Both a) and b d) Either a) or b)

18. The most commonly used measure of dispersion is

a) Coefficient of variation b) **Standard deviation**

c) Range d) Quartile deviation

19. Which one is an absolute measure of dispersion?

a) Standard deviation b) Mean deviation

c) Range d) **All these measures**

20. Coefficient of variation is

a) Absolute measure b) **Relative measure**

c) Both a) and b) d) None of these

21. The square of standard deviation is known as :

a) **Variance** b) Mean deviation

c) Standard deviation d) None of these

22. Which measure is based on only the central fifty per cent of the observations?

a) Mean deviation b) **Quartile deviation**

c) Standard deviation d) All these measures

23. Which measure of dispersion is the quickest to compute?

a) Standard deviation c) Mean deviation

b) Quartile deviation d) **Range.**

24. Which measure of dispersion is based on the absolute deviations only ?

a) Standard deviation c) Quartile deviation

b) **Mean deviation** d) Range.

25. Which measure of dispersion is based on all the observations ?

a) Mean Deviation c) Quartile deviation

b) Standard deviation **d) a) and b) but not c).**

26. The appropriate measure of dispersions for open – end classification is

a) Standard deviation **c) Quartile deviation**

b) Mean deviation d) All these measures.

CHAPTER 5

CORRELATION & REGRESSION

1. If for two variable x and y, the covariance, variance of x and variance of y are 40, 16 and 256 respectively, what is the value of the correlation coefficient ?

Solution : $cov(x,y) = 40$ $\sigma x = 4$ $\sigma y = 16$

$$r = \frac{cor(n,y)}{\sigma n.6y} = \frac{40}{4 \times 16} = 0.625$$

2. If cov (x, y) = 15, what restrictions should be put for the standard deviations of x and y ?

a) No restriction,
b) The product of the standard deviations should be more than 15.
c) The product of the standard deviations should be less than 15.
d) The sum of the standard deviations should be less than 15.

Solution : $r = \frac{cor(x,y)}{\sigma n.6y} = \frac{15}{\sigma x.6y}$ but $r < 1$ $\therefore \sigma x.\sigma y > 15$

3. If r = 0.6 then the coefficient of non-determination is

a) 0.4 c) 0.36
b) -0.6 d) 0.64.

Solution :
$r = 0.6$ $coefficient\ of\ non-determination = l - r^2 = l - (0.6)^2 = l - 0.36 = 0.64$

4. For the following data, the coefficient of rank correlation is:

Rank in botany:	1	2	3	4	5
Rank in chemistry:	2	3	1	5	4

(a) 0.93 (b) 0.4 (c) 0.6 (d) None

Solution :

Rank in botany:	1	2	3	4	5
Rank in chemistry:	2	3	1	5	4
D	-1	-1	2	-1	1
D^2	1	1	4	1	1

$N = 5$ $\sum d^2 = 8$

$$r = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6 \times 8}{5 * 24} = 0.6$$

5. If the sum of squares of difference of ranks, given by two judges A and B, of 8 students in 21, what is the value of rank correlation coefficient ?

a) 0.7 b) 0.65 c) -0.75 d) 0.8

Solution : $N = 8$ $\sum d^2 = 21$

$r = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6\times21}{8\times63}$ = 0.75

6. For 10 pairs of observations, No. of concurrent deviations was found to be 4. What is the value of the coefficient of concurrent deviation ?

a) $\sqrt{0.2}$ b) - $\sqrt{0.2}$ c) 1/3 d) -1/3.

Solution : $n = 10$ $m = n - 1 = 9$ $c = 4$ $r = \pm\sqrt{\pm\frac{(2c-m)}{m}} = \sqrt[-]{\frac{-(2\times4-9)}{9}} = \frac{-1}{3}$

7. If u + 5x = 6 and 3y – 7v = 20 and the correlation coefficient between x and y is 0.58 then what would be the correlation coefficient between u and v?

a) 0.58 c) -084.

b) -0.58 d) 0.84.

Solution : $b = \frac{-x}{u} = \frac{-5}{1} = -5$ $d = \frac{-y}{v} = \frac{-3}{-7} = \frac{3}{7}$

$rxy = 0.58$ $ruv = -rxy = -0.58$

8. If coefficient of correlation between x and y is 0.46. Find coefficient of correlation between x and $\frac{y}{2}$

(a) 0.46 (b) 0.92 (c) -0.46 (d) -0.92

Solution : $b = 1$ $d = \frac{1}{2}$ $r\,x, \frac{y}{2} = +rxy = 0.46$

9. If the relation between x and u is 3x + 4u + 7 = 0 and the correlation coefficient between x and y is -0.6, then what is the correlation coefficient between u and y?

a) -0.6 c) 0.6

b) 0.8 d) -0.8

Solution : $b = \frac{-x}{u} = \frac{-3}{4}$ d = 1 $ruy = -rxy = --0.6$ = 0.6

10. From the following data regarding the rainfall and the crop yield, estimated the yield when the rainfall I s 22 cms.

	Y Yield (In kgs.)	X Rainfall (In cms.)
Average	508.4	26.7
S.D.	36.4	4.6

Correlation co-efficient = 0.52

a) 32.65 b) **488.85** c) 466.6 d) 848.8

Solution: $\overline{x} = 26.7$ $\overline{y} = 508.4$

$\sigma x = 4.6$ $\sigma y = 36.4$

r=0.52 x=22

byx= r x $\frac{\sigma y}{\sigma x}$ = 0.52 x $\frac{36.4}{4.6}$ = 4.1147

y-508.4=4.1147(-4.7)

y= -19.3390+508.4=489.0609 ~ 488.85

13. From the following data regarding the rainfall and the crop yield, estimated the yield when the yield is 600 kg..

	Y Yield (In kgs.)	X Rainfall (In cms.)
Average	508.4	26.7
S.D.	36.4	4.6

Correlation co-efficient = 0.52

a) **32.65** b) 32 c) 36.6 d) 30.25

Solution: Y= 600 bxy = $0.52 \times \frac{4.0}{36.4}$ = 0.06571

$x - 26.7 = 0.657(91.6)$

$x = 6.0181 + 26.7$ =32.7181

14. If the regression line of y on x and that of x on y are given by y = 2x + 3 and 8x = y + 3 respectively, what is the coefficient of correlation between x and y?

a) 0.5 b) -1/ $\sqrt{2}$ c) -0.5 d) None of these.

Solution:

$y = 2x + 3$ $8x = y + 3$

$byx = 2$ $x = \frac{y}{8} + \frac{3}{8}$ compare with x = a + **b**Y then bxy = +1/8

$r = \pm\sqrt{byx \times bxy}$ = $\pm\sqrt{2 \times +1/8}$ = $+\sqrt{+0.25} = 0.5$

15. If 4y – 5x = 15 is the regression line of y on x and the coefficient of correlation between x and y is 0.75, what is the value of the regression coefficient of x on y?

a) 0.45 b) 0.9375 c) 0.6 d) None of these.

Solution:

$bxy = 0.75$ $Line\ of\ y\ on\ x\ is\ :\ 4y - 5x = 15$ $4y = 15 + 5x$

$4 = \frac{15}{4} + \frac{5}{4}x$ $byx = \frac{5}{4}$

$r = \pm\sqrt{byx \times bxy}$ $0.75 = \sqrt{5/4} \times \sqrt{bxy}$

$\frac{0.75}{1.1180} = \sqrt{bxy}$ $0.4489 = bxy$

16. Two random variables have the regression lines 3x+2y=26 and 6x+y=31.The coefficient of correlation is:

(a) -0.25 (b) 0.5 (c) -0.5 (d) 0.25

Solution:

$3x + 2y = 26$ $6x + y = 31$

$2y = 26 - 3x$ $\quad\quad$ $6x = 31 - 4$

$y = \frac{26}{2} - \frac{3x}{2}$ $\quad\quad$ x= $\frac{31}{6} - \frac{1}{6}x$

$byx = -3/2$ $\quad\quad$ $bxy = \frac{-1}{6}$

$r = \pm\sqrt{-3/2 \times -1/6}$ $\quad\quad$ $r = -0.5$

17. Given the regression equations as 3x + y = 13 and 2x + 5y = 20, which one is the regression equation of y on x?

a) 1st equation b) 2nd equation c) both a) and b) d) none of these.

Solution:

$2x + 5y = 26$ $\quad\quad$ $6x + y = 31$

$5y = 20 - 2x$ $\quad\quad$ $6x = 31 - 4$

$y = \frac{20}{2} - \frac{2}{2}x$ $\quad\quad$ x= $\frac{13}{3} - \frac{1}{3}y$

$byx = -2/5$ $\quad\quad$ $bxy = \frac{-1}{3}y$

$r = \pm\sqrt{-2/5 \times -1/3}$

$r = -0.3651$

18. If y = a + bx, then what is the coefficient of correlation between x and y?

a) 1 **c) 1 or -1 according as b > 0 or b < 0**

b) − 1 d) None of these.

19. If the lines of regression is a bivariate distribution are given by x+2y=5 and 2x+3y=8, then the coefficient of correlation is:

(a) 0.866 (b) -0.666 (c) 0.667 (d) -0.866

Solution:

$x + 2y = 5$ $\quad\quad$ $2x + 3y = 8$

$x = 5 - 2y$ $\quad\quad$ 3y= $8 - 2x$

bx$y = -2$ $\quad\quad$ y= $\frac{8}{3} - \frac{2}{3}x$

$r = \pm\sqrt{-2 \times -2/3}$

$r = -1.1547$

∴ =$\frac{1}{1.1547}$ = -0.8660

20. If the regression line of y and x and of x on y are given by 2x + 3y = -1 and 5x + 6y = -1 then the arithmetic means of x and y are given by

a) **(1, -1)** b) (-1, 1) c) (-1, -1), d) (2, 3)

Solution : Solve both equation simultaneously

21. Correlation analysis aims at :

a) Predicting one variable for a given value of the other variable.

b) Establishing relation between two variables.

c) Measuring the extent of relation between two variables.

d) Both b) and c).

22. Regression analysis is concerned with :

a) Establishing a mathematical relationship between two variables.
b) Measuring the extent of association between two variables
c) Predicting the value of the dependent variable for a given value of the independent variable.
d) Both a) and c)

23. Scatter diagram is considered for measuring :

a) Linear relationship between two variables
b) Curvilinear relationship between two variables.
c) Neither a) nor b).
d) Both a) and b).

24. If the plotted points in a scatter diagram lie from upper left to lower right, then the correlation is

a) Positive **c) Negative,**
b) Zero d) None of these.

25. The correlation between shoe-size and intelligence is :

a) **Zero** c) Negative
b) Positive d) None of these.

26. The correlation between the speed of an automobile and the distance travelled by it after applying the brakes is

a) Negative c) Zero
b) **Positive** d) None of these.

27. Two regression lines always intersect at the means.

a) **True** b) False c) Both d) None

28. The regression lines are identical if r is equal to

a) +1 b) -1 **c) $\pm$1** d) 0

29. What are the limits of the correlation coefficient ?

a) No limit c) 0 and 1, including the limits,
b) – 1 and 1 d) **-1 and 1, including the limits.**

30. For finding correlation between two attributes, we consider :

a) Person's correlation coefficient,
b) Scatter diagram,
c) **Spearman's rank correlation coefficient**,
d) Coefficient of concurrent deviations.

31. For finding the degree of agreement about beauty between two judges in a Beauty Contest, we use.

a) Scatter diagram c) Coefficient of correlation
b) **Coefficient of rank correlation,** d) Coefficient of concurrent deviation.

32. When we are not concerned with the magnitude of the two variables under discussion, we consider :

a) Rank correlation coefficient c) **Coefficient of concurrent deviation**
b) Product moment correlation coefficient d) a) or b) but not c).

CHAPTER 6

PROBABILITY

1. A box contains 6 black and 4 white balls. Two balls are drawn at random from it, the probability that both the balls are black is

a) 1/2 b) 1/3 c) 2/3 d) 1/4.

$Solution:$ $\frac{B}{6}\frac{W}{4}$ $= {}^{6}c_2 / {}^{10}C_2 = 15/45 = 1/3$

2. A box contains 6 black and 4 white balls. three balls are drawn at random from it, probability that there are 2 white & one black ball is

a) 1/15 b) 1/5 c) 2/15 d) 4/15

Solution : B W

$$\frac{6}{1}\frac{4}{2} = {}^{6}c_1 \times {}^{4}c_2 / {}^{10}C_3 = \frac{6\times6}{120} = \frac{36}{120} = \frac{3}{10}$$

3. The probability that a leap year will have 53 Sundays is :

a) 1/7 b) 2/7 c) 3/7 d) 1/53

Solution : P(53 Sunday in non-leap year)

365

52 1

$\frac{\times 7}{364}$ single day

Full weeks Sunday = $1/7$

P(53. Sundays in leap Year) = 366

52 2days

$\frac{\times 7}{364}$ Sunday = $2/7$

4. A speaks truth in 60% of the cases and B in 90% of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact?

(a) 36% (b) 42% (c) 54% (d)None of these.

Solution : P(A)= 0.6 P(A′)=0.4 P(B)= 0.9 P(B′)=0.1

P(A & B′) + P(B & A′0 = $[0.6 \times 0.1] + [0.9 \times 0.4]$ =0.06 + 0.36 =0.42 X 100 =42%

5. Three persons A, B and C aim a target. The probabilities of their hitting the target are respectively 2/3, 1/4,1/2. What is the probability that the target will be hit ?

a) 1/8 b) 3/8 c) 5/8 d) 7/8

Solution : P(A) = 2/3 P(B) = ¼ P(C) = 1/2

P(A′) = 1/3 P(B′) = ¾ P(C′) = 1/2

P(Target will be hit)= 1 – (target will not be hit) = 1 – P(A' * B' * C') = 1- (1/3x 3/4x ½)
= 1- (1/8) = 7 / 8

6. An example of statistics is given to three students A, B and C. Their probabilities of solving the example correctly are respectively 1/2 , 3/4, ¼ the probability that the example will be solved is

a) 20/32 b) 27/32 c) 28/32 d) 29/32

Solution : P(A) = 1/2 $P(A')$ = ½ P(B) = ¾ $P(B')$ = 1/4

P(C) = ¼ $P(C')$ = ¾

P (solving the problem) = 1 - P(not solving problem)= 1 –[½ x ¼ x ¾]
= 1 – 3/32
= 29/32

7. The present age of a person A is 35. The odds in favour of his living upto the age of 65 is 3 : 2. The age of another person B is 40 at present. The odds against his living upto the age of 70 is 4 : 1. The probability that atleast one of them will be alive after 30 years is

a) 17/30 b) 17/25 c) 18/72 d) 7/25

Solution : P(A) = 3/5 $P(A')$ = 2/5

P(B) = 4/5 $P(B')$ = 4/5
P (Atleast one will be alive) = I-P(no one alive) = 1- p (A' * B') = I-[2/5x 4/5]
=I-8/25
=17/25

8. For a 60 years old person living upto the age of 70, it is 7 : 5 against him and for another 70 years old person surviving upto the age of 80, it is 5 : 2 against him. The probability that only one of them will survive for 10 years more is :

a) 15/42 b) **39/84** c) 49/84 d) 40/84

Solution : P(A) = 5/12 $P(A')$ = 7/12

P(B) = 2/7 $P(B')$ = 5/7

$P(AB')+P(BA')$ $=\frac{5}{12}\times\frac{5}{7}+\frac{2}{7}\times\frac{7}{12}=\frac{35}{84}+\frac{14}{84}=\frac{39}{84}$

9. A and B are mutually exclusive events of an experiment. If P(not A)=0.65, P(A ∪ B)=0.65 and P (B) = P, then the value of p is

(a) 0.45 **(b) 0.30** (c) 0.25 (d) None of these.

Solution : $P(A^1)$ = 0.65, P(AUB)= 0.65, P(B)= P, P(A) = 0.35

A & B are mutually exclusive then $P(A\cap B)=0$
:. $P(AUB)= P(A) + P(B) – P(A\cap B)$
0.65 = 0.35 + P(B)-0 p(B) = 0.30

10. Given that P(A) = 1/3, P(B) – 1/4, P (A | B) = 1/6, the probability P(B|A) is equal to :

a) 4/8 b) 3/8 c) 2/8 d) 1/8

Solution : P(A)=1/3 P(B)=1/4, P(A/B)=1/6, P(B/A)=?

P(A/B)= $\frac{P(A\cap B)}{P(B)}$:. 1/6= $\frac{P(A\cap B)}{1/4}$

1/6 x ¼ = P(A∩B) 1/24 = P(A∩B)

P(B/A)= $\frac{P(A\cap B)}{P(A)}$ = $\frac{1/24}{1/3}$ = 3/24 = 1/8

11. Given that P(A) = 1/3, P(B) = ¾ and P (AUB) = 11/12, the probability, P (B/A) is

a) 1/6 b) 4/9 c) 1/2 d) 1/4

Solution :

P(A) = 1/3, P(B) =3/4, P(AUB) = 11/12, P(B/A)=?

P(AUB) = P(A) + P(B) – P (A∩B)
11/12 = 1/3 + ¾ - P(A∩B)
11/12 = 13/12- P(A∩B)
11/12-13/12 =-P(A∩B)
-2/12 = -P (A∩B)
:. P(A∩B) = 2/12

P(B/A) = $\frac{2/12}{1/3}$ = $2/12 \times 3/1 = 2/4 = 1/2$

12. For a random variable x, E(x) = 2, the value of the E (2x + 3) is

a) 7 b) 5 c) 4 d) 3

Solution : mean = E(x) =2 E(2x+3) =[2(2)+3] = 7

13) From a pack of cards, two are drawn, the first being replaced before the second is drawn. The chance that the first is a diamond and the second is king is :

a) $\frac{1}{52}$ b) $\frac{3}{2704}$ c) $\frac{4}{13}$ d) $\frac{3}{52}$

Solution: $\frac{13_{c_1}\times 4_{c_1}}{52_{c_1}\times 52_{c_1}} = \frac{52}{2704}$ = 1/52

14) The theory of compound probability states that for any two events A and B :

a) P (A∩ B) = P(A) x P(B)
b) P (A ∩ B) = P(A) x P(B/A)
c) P (AU B) = P(A) x P(B/A)
d) P (A ∪ B) = P(A) + P(B) – P (A ∩ B)

15) If P (A ∩B) = P (A) x P (B), then the events are :

a) Independent events.
b) Mutually exclusive events
c) Exhaustive events
d) Mutually inclusive events.

CHAPTER

INDEX NUMBER

1. Find the index number by the method of relatives (using arithmetic mean)from the following data

Commodity	Base Price	Current Price
Rice	35	42
Wheat	30	35
Pulse	40	38
Fish	107	120

a. 110 b. 115 c. 120 d. 125

Solution :

Po	Pn	Pn/Po
35	42	1.2
30	35	1.66
40	38	0.95
107	120	1.121
212	235	4.4381

Pon $= \frac{\sum \frac{Pn}{Po} \times 100}{N} = \frac{4.4381}{4} \times 100 = 110.95$

Refer data for the Question

Commodity	1979		1980	
	Price in Rs.	Quantity In Kg.	Price in Re.	Quantity
A	20	8	40	6
B	50	10	60	5
C	40	15	50	10
D	20	20	20	15

2. Which of the following represent Paasche's price index number

a. 125.23 b. 124.70 c. 124.96 d. 125.95

Po	Qo	Pn	Qn	PnQo	PoQo	PnQn	PoQn
20	8	40	6	320	120	240	120
50	10	60	5	600	250	300	250
40	15	50	10	750	400	500	400
20	20	20	15	400	400	300	300
				2070	1660	1340	1070

Solution : Paasche's $= \frac{\sum Pn \times Qn}{\sum Po \times Qn} \times 100 = \frac{1340}{1070} \times 100 = 125.95$

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3. Which of the following represent Laspeyer's Price index Number

a. 125.23 b. 124.70 c. 124.96 d. 125.95

Solution : $\frac{\sum Pn \times Qn}{\sum Po \times Qu} \times 100 = \frac{2070}{1660} \times 100 = 124.698$

4. Which of the following represent Fisher's Price index Number

a. 125.23 b. 124.70 c. 124.96 d. 125.95

Solution = $\sqrt{124.70 \times 125.95} = 125.32$

5. Which of the following represent Marshall Edgeworth Price Index Number

a. 125.23 b. 124.70 c. 124.96 d. 125.95

$Solution$: $\frac{Qn+Qo}{2}$ Pn($\frac{Qn+Qo}{2}$) Po($\frac{Qn+Qo}{2}$)

7	280	
7.5	450	
12.5	625	
17.5	350	--------
	1705	1365

$\frac{1705}{1365} \times 100 = 124.90$

6. Laspeyre's and Paasche's method _____ time reversal test

a) Satisfy **b) Do not satisfy** c) Are d) Are not.

7. There is no such thing as unweighted index numbers

a) **False** b) True c) Both d) None.

8. Theoretically, G.M. is the best average in the construction of index nos. but in practice, mostly the A.M. is used.

a) False **b) True** c) both d) none

9. Laspeyre's or Paasche's or the Fisher's ideal index do not satisfy :

a) Time Reversal Test **c) Circular Test**

b) Unit Test d) None.

10. The test of shifting the base is called :

a) Unit Test **c) Circular Test**

b) Time Reversal Test d) None

11. The no. of test of Adequacy is :

a) 2 b) 5 c) 3 **d) 4**

12. The best average for constructing an index numbers is

a) Arithmetic Mean **c) Geometric Mean**

b) Harmonic Mean d) None of these.

13. The time reversal test is satisfied by

a) **Fisher's index number,** c) Laspeyre's index number

b) Paasche's index number d) None of these.

14. Paasche index is based on

a) Base year quantities. c) Average of current and base year.

b) **Current year quantities.** d) None of these.

15. Fisher's ideal index number is

a) The Median of Laspeyre's and Paascher's index number.

b) The Arithmetic Mean of Laspeyre's and Paasche's.

c) The Geometric Mean of Laspeyre's and Paasche's

d) None of these.

16. Net monthly salary of an employee was Rs. 3000 in 1980. The consumer price index number in 1985 is 250 with 1980 as base year. If he has to be rightly compensated, then the Dearness Allowance to be paid to the employee is :

a) Rs. 4,200 b) Rs. 4,500 c) Rs. 4,900 d) Rs. 7,500.

Solution : Dearress Allowance

	1980		1985
Index Number	100		250
3000	x	7500	
7500-3000=4500			

17. P_{10} is the index for time :

a) **0 on 1** b) 1 on 0 c) 1 on 1 d) 0 on 0

18. Shifted Price Index

$$\frac{Original\ Price\ Index}{=Price\ index\ of\ the\ year\ on\ which\ x\ 100: it\ has\ to\ be\ shifted.}$$

a) **True** b) False c) Partly True d) Partly False.

19. Consumer price index is commonly known as :

a) Chain Based Index c) Wholesale price index

b) Ideal Index d) **Cost of living index.**

20. Wholesale Price Index (WPI) is given by :

a) Marchall – Edgeworth Index c) Paasche's Index

b) Laspeyre's Index d) None of the above.

CHAPTER 9

THEOROTICAL DISTRIBUTION

1. What is the probability of making 3 correct guesses in 5 True – False answer type questions ?

a) 0.4156 c) 0.3125

b) 0.32 d) 0.5235

Solution: N= 5 x= no of correct guesses = 0,1,2,3,4,5

P=1/2, q=1/2

P(3 correct guess)= P(x=3) = ${}^5C_3 \times [\frac{1}{2}]^3 \times [\frac{1}{2}]^2$ = 10x0.125x0.25 = 0.3125

2. The Interval (μ - 38, μ + 38) covers :

a) 95% area of normal distribution

b) 96% area of normal distribution

c) 99% area of normal distribution

d) All but 0.27% area of a normal distribution.

3. The overall percentage of failure in a certain examination is 0.30. What is the probability that out of a group of 6 candidates at least 4 passed the examination ?

a) 0.74 c) 0.59

b) 0.71 d) 0.67.

Solution: P(x=4,5,6)

$P(x = 4) + P(x = 5) + P(x = 6)$

${}^6c_4$ x $(0.7)^4$ x $(0.3)^2$ + ${}^6c_5$ x $(0.7)^3$ x $(0.3)^1$ + ${}^6C_6$ x (0.7)

=0.3241+0.3025+0.1176 = 0.7443

4. A manufacturer, who produces medicine bottles, finds that 0.1% of the bottles are defective. The bottles are packed in boxes containing 500 bottles. A drug manufacturer buys 100 boxes from the producer of bottles. Using Poisson distribution, find how many boxes will contains at least two defectives :

(Given $e^{-0.5}$ = 0.6065)

a) 7 b) 13 c) 9 d) 11

Solution: N=np=500x0.1%=0.5

P(at least 2 are detective) = P(x=2,3,4,5…….∞) = 1-P(x=0)-P(x=1)

$$= 1 - \frac{e^{-0.5} \times 0.5^0}{0!} - \frac{e^{-0.5} \times 0.5^0}{1!}$$

$$= 1 - 0.6065 - 0.30325 = 0.69675 = 0.09025 \times 100 = 9\%$$

5. Suppose that weather records show that on an average 5 out of 31 days in October are rainy days. Assuming a binomial distribution with each day of October as an independent trial, then the probability that the next October will have at most three rainy days is :

a) 0.4403 b) 0.2403 c) 0.3403 d) None.

Solution: P(at most 3 rainy days)

x= No of rainy days =0,1,2,……..31

n=31 p=5/31 = 0.1612 q= 26/31 = 0.8388

P(x=0,1,2,3)

$={}^{31}c_0 \times 0.1612^0 \times 0.8388^{31} + {}^{31}c_1 \times 0.1612^1 \times 0.8388^{30}$

$+{}^{31}c_2 \times 0.1612^4 \times 0.8388^{29} + {}^{31}c_3 \times 0.1612^3 \times 0.8388^{28}$

= 1x1x0.0042+31x0.1612x

6. If 5% of the families in Kolkata do not use gas as a fuel, what will be the probability of selecting 10 families in a random sample of 100 families who do not use gas as fuel?

(Given : $e^{-5} = 0.0067$)

a) 0.038 b) 0.026 c) 0.048 d) 0.018

Solution: P(10 Families Who do not use gas) = P(x = 10)

m= np =100x0.05 = 5

P(x=10) $= \frac{e^{-m} \times m^x}{x!} = \frac{0.0067 \times 5^{10}}{10!} = \frac{65429.6875}{3628806} = 0.0180$

7. If 15 dates are selected at random, then the probability of getting two Sundays is:

a) 0.29 b) 0.99 c) 0.49 d) 0.39

Solution: n=15, P=1/7, q=6/7

x=no of Sundays

P(2 Sundays) = P (x =2) $F(x) = {}^{15}C_2 \times [\frac{1}{7}]^2 \times [\frac{6}{7}]^{13} = 0.288$

8. In a certain manufacturing process, 5% of the tools produced turn out to be defective. Find the probability that in a sample of 40 tools, at most 2 will be defective :

(Given : $e^{-2} = 0.135$)

a. 0.555 b. 0.932 c. 0.785 d. 0.675.

Solution: P=0.05 use formula for poisson distribution

np= m 40 x 0.05=2

P(at most 2)= P(x=0) +P(x=1) +P(x=2) = 2.7182

9. Examine the validity of the following : Mean and standard Deviation of a binomial distribution are 10 and 4 respectively.

a) **Not valid** b) Valid c) Both a & b) d) Neither a) nor b).

Solution : mean = 10 variance = 16 but mean is always greater than variance

10. An experiment succeeds twice as often as it fails. What is the probability that in next five trials there will be at least three successes ?

a) $\frac{33}{81}$ b) $\frac{46}{81}$ c) $\frac{64}{81}$ d) $\frac{25}{81}$

Solution: P=2q P=2(1-P) P=2-2P 3P=2

P=2/3 .: q=1/3 n=5

P(x=3,4,5) $={}^5C_3 \times [\frac{2}{3}]^3 \times [\frac{1}{3}]^2 + {}^5C_4 \times [\frac{2}{3}]^4 \times [\frac{1}{3}]^1 + {}^5C_5 \times [\frac{2}{3}]^5 \times [\frac{1}{3}]^0$ = 64 / 81

11. In Poisson Distribution, probability of success is very close to :

a) -1 **b) 0** c) 1 d) Non

12. If the mean of a poisson variable X is 1, what is P (x = at least one)?

a) 0.456 b) 0.821 c) 0.632 d) 0.254

Solution : m =1 P (X = atleast one) = P(X=1,2,3,4…) = 1-P(x=0)

$$=1-\ \frac{e^{-m}\times m^{x}}{x!}\ =1-\frac{e^{-1}\times(1)^{0}}{0!}\ \ 1-\frac{1}{2.7182}\times\frac{1}{1}$$ = 80.3632 = 1-0.3678 = 0.632

13. What is the probability of getting 3 heads if 6 unbiased coins are tossed simultaneously?

a) 0.3125 b) 0.25 c) 0.6875 d) 0.50

Solution : P (X=3) = ${}^{6}C_{3}\times(0.5)^{3}\times(0.5)^{3}$ = 20 x 0.125 x 0.125 = 0.3125

14. In a poisson distribution P (x = 0) = P (X = 2). Find E (x).

a) $\sqrt{2}$ b) 2 c) -1 d) 0

Solution : P(x=0) = P(x=2)

$$\frac{e^{-m}\times m^{0}}{0!}=\frac{e^{-m}\times m^{2}}{2!}$$

$$\frac{1}{1}=\frac{m*m}{2}$$ $m^2=2$

m= $\sqrt{2}$ = mean= E (x)

15. For binomial distribution E(x) = 2, V (x) = 4/3. Find the value of n.

a) 3 b) 4 c) 5 d) 6

Solution : E(x) =2 = np

V(x)=4/3

Npq=4/3 put np= 2 2q=4/3

$q=^{4}/_{3}\times 2$ = 2/3 p = 1/ 3 n=6

16. What are the parameters of binomial distribution?

a) n b) p c) **Both n and p** d) None of these.

17. If standard deviation of a poisson distribution is 2, then its

a) Mode is 2
b) Mode is 4
c) Modes are 3 and 4
d) Modes are 4 and 5

18. The area under the Normal curve is :

a) **1** b) 0 c) 0.5 d) -1